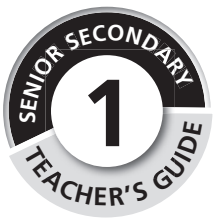
Excellence in
Mathematics



Published by Cambridge University Press
Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

Distributed in Nigeria by Cambridge University Press Nigeria Ltd
252E Muri Okunola Street, Victoria Island, Lagos State, Nigeria

Cambridge University Press & Assessment is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

First published 2016

Printed in India by Multivista Global Pvt Ltd.

ISBN 978-1-316-60873-9 Paperback

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Cover image: Shutterstock Veronica By/Shutterstock

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Introduction

The purpose of the curriculum

The main objectives of the Mathematics curriculum are to prepare the students to:

- acquire the mathematical literacy necessary to functioning in an information age
- cultivate the understanding and application of mathematical concepts and skills necessary to thrive in the ever-changing technological world
- develop the essential skills of problem solving, communication, reasoning and connection within the study of Mathematics
- take advantage of the numerous career opportunities provided by Mathematics
- prepare for further studies in Mathematics and other related fields.

The role of the teacher

One of the principal duties of a Mathematics teacher is to prepare and present good lessons to students. In order to do this, the teacher needs to:

- be as well informed as possible on the scheme of work of the subject
- know the aims and objective of each topic
- select appropriate content material
- decide on the best methods of presentation such as group work, worksheets, question–answer sessions, debate, etc.
- keep informed about social and environmental issues and other current news in Nigeria and the rest of the world
- through innovative teaching approaches encourage learning that will promote creativity and critical thinking.

To be effective in presentation, the teacher should prepare a written or typed plan for each lesson. This must include aims, objectives, resources, time frames, content for the lesson, activities, homework, assessment, and ideas for additional worksheets to cater for students requiring extension or learning support (remediation).

Prepare each topic in advance. It is your responsibility as a Mathematics teacher to involve your students actively in the learning process. It is a proven fact that students learn far more by *doing* than by *listening*.

Mathematics involves being curious and asking questions. Wherever possible ask questions to engage the students, to encourage independent thought processes, and to develop problem-solving skills. Start your lessons by asking the students to write down answers to questions related to your lesson (approximately five). This will settle them into the lesson.

You can use different types of questions in your lessons:

- **diagnostic** questions enabling you to determine prior knowledge on the topic
- for **consolidation** of challenging concepts during the lesson
- for **stimulation** of interest in the subject
- for **concluding** the lesson.

Concluding questions will assist you in finding out whether students have understood the concepts and terminology of the lesson. It will also highlight any areas that they need to revise at home or for you to revisit in the next lesson.

It is best to ensure that you do not appear to have favourites in the class, so devise a system to ask questions fairly, but be careful not to embarrass weak students if they cannot answer.

How to use the scheme of work

A scheme of work is defined as the part of the curriculum that a teacher will be required to teach in any particular subject. Its primary function is to provide an outline of the subject matter and its content, and to indicate how much work a student should cover in any particular class. A scheme of work allows teachers to clarify their thinking about a subject, and to plan and develop particular curriculum experiences that they believe may require more time and attention when preparing lessons. The criteria all teachers should bear in mind when planning a scheme of work are continuity in learning and progression of experience. You can add your own notes to the scheme of work provided on pages vii to x.

The scheme of work is sequential. The sequence of the scheme of work is aligned with the Student's Book. Do not be tempted to jump around. Rather spend time carefully planning the term to ensure that you adhere to the scheme of work.

The year is divided into three terms, and each term is divided into 13 weeks. There are 6 topics in Term 1, 5 topics in Term 2 and 4 topics in Term 3. The end of term allows time for revision and an examination. This time frame may vary depending on the planning of your particular school.

Your management of the class will have an enormous influence on your ability to adhere to the time frames. Focus on effective discipline strategies. You will have less discipline issues if you are: punctual, well prepared, follow a plan (write this on the board at the start of the lesson), keep your word (don't make empty threats) and consistently adhere to rules.

A teacher of Mathematics is a professional instructor who facilitates, promotes and influences students to achieve the outcomes of the scheme of work. It is the wish of the authors that the students will, at the end of each course in the series, attain a level of Mathematics proficiency that will equip them for future studies in this field.

Scheme of work

Term 1

Topic	Lesson objectives	Student's Book pages
1. Revision	Students should be able to: • draw on prior knowledge of mathematics learnt in JSS3	1–16
2. Number base systems	Students should be able to: • convert numbers from other bases to base 10 • convert decimal fractions from other bases to base 10 • convert from one base to another • perform basic operations in different bases (other than base 2) • apply number bases to computer programming	17–37
3. Modular arithmetic	Students should be able to: • explain the concept of modular arithmetic • carry out addition, subtraction and multiplication in modular arithmetic • apply modular arithmetic in daily life	38–46
4. Indices and standard form	Students should be able to: • approximate numbers • convert numbers to standard form notation • solve problems on standard form • apply the laws of indices • identify indices as short hand of the standard form • solve problems using indical equations	47–68
5. Logarithms	Students should be able to: • explain the relationship between indices and logarithms • define a logarithm • convert between index form and logarithm form • use the graph of $y = 10^x$ to determine logarithm values • find logarithms and antilogarithms of numbers greater than one • use logarithm tables in calculations • apply logarithms to solve problems relating to capital markets	69–81

Topic	Lesson objectives	Student's Book pages
6. Linear equations and variations	Students should be able to: • change the subject of a linear equation • solve problems involving direct, inverse, joint and partial variation • apply variation to physical laws and real life situations	82–97

Term 2

Topic	Lesson objectives	Student's Book pages
7. Quadratic equations	Students should be able to: • solve quadratic equations using factorising and completing the square • derive and use the quadratic formula • draw the quadratic graph and use it to read roots and turning point • form a quadratic equation given the roots • apply the quadratic equation to real life situations	98–116
8. Sets	Students should be able to: • explain a mathematical set and set terminology • use set builder notation • describe and use the various types of sets • draw, interpret and use Venn diagrams • find the union and intersection of sets • state or represent the complement of a set • use Venn diagrams to solve problems involving sets	117–138
9. Mensuration: Circles	Students should be able to: • draw on prior knowledge of parts and properties of circles • calculate the length of arc of a circle • calculate the perimeter of sectors and segments • determine the area of circles, sectors and segments	139–149

Topic	Lesson objectives	Student's Book pages
10. Trigonometry	Students should be able to: • find the sine, cosine and tangent of acute angles in right-angled triangles • determine the length of a chord using trigonometric ratios • derive the trigonometric ratios of the special angles • simplify expressions involving the special angles • work with ratios on the Cartesian plane • relate sine and cosine ratios to the unit circle • draw graphs of sine and cosine for angles from 0° to 360° • apply the trigonometric ratios to practical problems • solve practical problems on angles of elevation and depression, bearing and distance	150–175
11. Logical reasoning	Students should be able to: • identify open and closed simple statements • differentiate between simple true and false statements • state the negation of a simple statement • distinguish between simple and compound statements • identify and use conjunctions, disjunctions, implications and bi-implications • construct truth tables for each of the five logical operations • write the truth values of compound statements involving any of the five logical operations	176–188

Term 3

Topic	Lesson objectives	Student's Book pages
12. Mensuration: Solids	Students should be able to: • state the properties of a cube, cuboid, cylinder, triangular prism, cone and pyramid • determine the total surface area of these solids • determine the volume of these solids • determine the surface area and volume of a frustum • determine the surface area and volume of compound shapes	189–206

Topic	Lesson objectives	Student's Book pages
13. Constructions	Students should be able to : • construct and bisect line segments and angles • construct angles of 22.5°, 30°, 45°, 60°, 90°, 120°, 135° and 150° • construct triangles and quadrilaterals • work with the loci of moving points	207–218
14. Triangles, parallel lines, parallelograms	Students should be able to: • distinguish between different types of angles and the relationships between them • set out a formal proof • prove that the sum of the angles of a triangle is 180° • prove that any exterior angle of a triangle is equal to the sum of the two interior opposite angles • prove the intercept theorem • prove riders and solve problems on angles on parallel lines, congruent triangles, parallelograms and other polygons	219–242
15. Statistics	Students should be able to: • calculate the mean, median, mode, and range of ungrouped data • summarise ungrouped data in frequency tables • group data in classes using class intervals • determine class boundaries and midpoints of class intervals • summarise grouped data in frequency tables • estimate the median of grouped data • identify the modal class of grouped data • represent data in pie charts, line graphs, bar graphs, histograms and frequency polygons	243–268
16. Revision and exam practice	Students will carry out practice problems on: • number and numeration • algebraic processes • geometry • statistics	269–281

TOPIC 1

Revision

Introduction

This first topic is a revision of the main concepts covered in Mathematics in JSS3. We have not attempted to cover all the work that was done in JSS3 but instead have focused on the concepts that will be revisited in SS1. The material is arranged by theme and follows a logical progression through the curriculum.

Preparation

Prepare posters that show the main formulae used in this topic and display these around your classroom for your students to refer to.

Before presenting this topic to your class, give careful thought to how you want to work through this revision material. All the exercises can and should be used at your own discretion. In some cases, you might choose to set your students selected questions only or even to skip an exercise altogether. In other cases, you might prefer to postpone some sections of this revision to a later stage in the year, if you feel that your students will benefit more by doing the revision just before the relevant section of the work is covered in SS1.

Introduction for the students

Explain to your class how you have decided to work through this revision material. Some sections will benefit by a class discussion, some sections can be given as homework and some sections can be given as group or pair work. By varying your methodology, you will hold the interest of your students.

Answers

Exercise 1.1

1. a) $101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 4 + 0 + 1$
$= 5_{10}$

b) $10\ 110_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
$= 16 + 0 + 4 + 2 + 0$
$= 22_{10}$

2. a) $25_{10} = 16 + 8 + 1$
$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 11\ 001_2$

b) $83_{10} = 64 + 16 + 2 + 1$
$= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1$
$+ 1 \times 2^0$
$= 1\ 010\ 011_2$

3. a) $201_3 = 2 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$
$= 18 + 0 + 1$
$= 19_{10}$

b) $3\ 120_4 = 3 \times 4^3 + 1 \times 4^2 + 2 \times 4^1 + 0 \times 4^0$
$= 192 + 16 + 8 + 0$
$= 216_{10}$

c) $3\ 412_5 = 3 \times 5^3 + 4 \times 5^2 + 1 \times 5^1 + 2 \times 5^0$
$= 375 + 100 + 5 + 2$
$= 482_{10}$

d) $674_8 = 6 \times 8^2 + 7 \times 8^1 + 4 \times 8^0$
$= 384 + 56 + 4$
$= 444_{10}$

4. a) $137_{10} = 81 + 2 \times 27 + 2$
$= 1 \times 3^4 + 2 \times 3^3 + 0 \times 3^2 + 0 \times 3^1 + 2 \times 3^0$
$= 12\ 002_3$

b) $137_{10} = 2 \times 64 + 8 + 1$
$= 2 \times 4^3 + 0 \times 4^2 + 2 \times 4^1 + 1 \times 4^0$
$= 2\,021_4$

c) $137_{10} = 125 + 2 \times 5 + 2$
$= 1 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 2 \times 5^0$
$= 1\,022_5$

d) $137_{10} = 2 \times 64 + 8 + 1$
$= 2 \times 8^2 + 1 \times 8^1 + 1 \times 8^0$
$= 211_8$

Exercise 1.2

1. a) If speed is constant, then distance = $k \times$ time, so distance and time are directly proportional.
b) If distance is constant, then time = $\frac{k}{\text{speed}}$, so time and speed are inversely proportional.

2. a) $y = 5x$
b) x and y are directly proportional.
c) The constant of proportionality is 5.

3. a) $y = \frac{₦12\,000}{x}$
b) x and y are inversely proportional.
c) The constant of proportionality is ₦12 000.

4. a) $\frac{5}{4}$ **b)** $\frac{6}{1}$ or 6 **c)** $-\frac{13}{5}$

d) $\frac{2}{17}$ **e)** $\frac{1}{3}$ **f)** $-\frac{4}{1}$ or -4

Exercise 1.3

1. If the interest rate is constant and the initial value of the investment is fixed, then SI = kn, where k is a constant. So SI and n are directly proportional.

2. a) $\text{SI} = \frac{n\text{P}r}{100}$

$= \frac{500\,000 \times 9 \times 6}{100}$

$= ₦270\,000$

So the interest earned will be ₦270 000.

b) $A = P(1 + \frac{r}{100})^n$

$= 500\,000(1 + \frac{8.5}{100})^6$

$= ₦815\,733.75$

Interest = A – P

$= ₦815\,733.75 - ₦500\,000$

$= ₦313\,733.75$

So the interest earned will be ₦313 733.75.

c) Jire should choose Bank B, because she will earn more interest there.

3. Jire should borrow the money from Bank A, because she will pay less interest there.

4. $A = P\left(1 - \frac{r}{100}\right)^n$

$\therefore \frac{P}{2} = P\left(1 - \frac{r}{100}\right)^n$

$\therefore \left(1 - \frac{r}{100}\right)^n = \frac{1}{2}$

$\therefore \left(1 - \frac{12.5}{100}\right)^n = \frac{1}{2}$

$\therefore (1 - 0.125)^n = \frac{1}{2}$

$\therefore 0.875^n = 0.5$

Try $n = 3$: $0.875^3 = 0.66992\ldots$

Try $n = 5$: $0.875^5 = 0.51290\ldots$ which is very close!

Try $n = 6$: $0.875^6 = 0.44879\ldots$

So it will take 6 years for the value of the tractor to halve.

5. $A = P\left(1 + \frac{r}{100}\right)^n$

$= 400\left(1 + \frac{8.5}{100}\right)^{10}$

$= ₦904.39$

Exercise 1.4

1. a) $\frac{22}{7}$ is rational, because it is in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

b) -0.25 is rational, because it terminates.

c) π is irrational, because it does not terminate and it also does not recur.

d) $\sqrt{\frac{4}{9}} = \frac{2}{3}$, which is rational, because it is in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

e) 0 is rational, because it terminates.

f) $0.\dot{6}$ is rational, because it recurs.

g) $-\frac{4}{9}$ is rational, because it is in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

h) 1.092 is rational, because it terminates.

i) $\sqrt{144} = 12$, which is rational, because it terminates.

j) $\sqrt{5}$ is irrational, because it does not terminate and it also does not recur.

2. a) $C = 2\pi r$

$\therefore 2 \times \pi \times 6 = 37.7$

$\therefore \pi = \frac{37.7}{12}$

$= 3.141\dot{6}$

b) The number $3.141\dot{6}$ is rational, because it recurs, but this is only an approximation of π, because the measurement of the circumference of the circle was rounded off to the nearest millimetre.

Exercise 1.5

1. a) $36x^3 - 9y^2 + 18xy = 9(4x^3 - y^2 + 2xy)$

b) $2x^2 - 200 = 2(x^2 - 100)$

$= 2(x - 10)(x + 10)$

c) $-6x + 16 - x^2 = -x^2 - 6x + 16$

$= -(x - 2)(x + 8)$

d) $x - y - ay + ax = (x + ax) - (y + ay)$

$= x(1 + a) - y(1 + a)$

$= (1 + a)(x - y)$

e) $3x^3y^2 - 6xy^2 = 3xy^2(x^2 - 2)$

f) $a(x-1)-b(x-1)+c(1-x)=(x-1)(a-b-c)$

g) $x^2-7x-60=(x-12)(x+5)$

h) $2x^2+14x-36=2(x^2+7x-18)$
$=2(x+9)(x-2)$

i) $25x^2-81y^2=(5x+9y)(5x-9y)$

j) $2x^2+xy-8xy-4y^2=(2x+y)(x-4y)$

2. $57^2-53^2=(57+53)(57-53)$
$=110(4)$
$=440$

Exercise 1.6

1. $\frac{x}{3}+\frac{x}{5}=4$

$\therefore 5x+3x=60$

Multiply all terms by LCM 15.

$\therefore 8x=60$

$\therefore x=\frac{15}{2}$

2. $5-\frac{x}{3}=\frac{x}{2}$

$\therefore 30-2x=3x$

$\therefore -2x-3x=-30$

Multiply by LCM 6.

$\therefore -5x=-30$

$\therefore x=6$

3. $\frac{5x-1}{3}=\frac{x+4}{4}$

$\therefore 4(5x-1)=3(x+4)$ Multiply by LCM 12.

$\therefore 20x-4=3x+12$

$\therefore 20x-3x=12+4$

$\therefore 17x=16$

$\therefore x=\frac{16}{17}$

4. $\frac{3x-4}{2}=\frac{4x+5}{3}-\frac{x-1}{6}$

$\therefore 3(3x-4)=2(4x+5)-(x-1)$ Multiply by LCM 6.

$\therefore 9x-12=8x+10-x+1$

$\therefore 9x-8x+x=10+1+12$

$\therefore 2x=23$

$\therefore x=\frac{23}{2}$

5. $1 - \frac{1}{2}(x + 3) = \frac{3}{4}(2x - 1)$

$\therefore 4 - 2(x + 3) = 3(2x - 1)$ Multiply by LCM 4.

$\therefore 4 - 2x - 6 = 6x - 3$

$\therefore -2x - 6x = -3 - 4 + 6$

$\therefore -8x = -1$

$\therefore x = \frac{1}{8}$

6. $6 = \frac{18}{4 - x}$

$\therefore 6(4 - x) = 18$ Multiply by LCM $(4 - x)$.

$\therefore 24 - 6x = 18$

$\therefore -6x = -6$

$\therefore x = 1$

7. $\frac{x + 1}{x - 2} = \frac{x - 1}{x - 3}$

$\therefore (x + 1)(x - 3) = (x - 1)(x - 2)$ Cross-multiply.

$\therefore x^2 - 2x - 3 = x^2 - 3x + 2$

$\therefore -2x + 3x = 2 + 3$

$\therefore x = 5$

8. $\frac{2x - 4}{x + 1} = \frac{2x + 1}{x + 2}$

$\therefore (2x - 4)(x + 2) = (2x + 1)(x + 1)$ Cross-multiply.

$\therefore 2x^2 - 8 = 2x^2 + 3x + 1$

$\therefore -3x = 8 + 1$

$\therefore x = -3$

Exercise 1.7

1. $V = lbh$

$\therefore lbh = V$

$\therefore h = \frac{V}{lb}$

2. $C = 2\pi r$

$\therefore 2\pi r = C$

$\therefore r = \frac{C}{2\pi}$

3. $A = P\left(1 + \frac{r}{100}\right)^n$

$\therefore P\left(1 + \frac{r}{100}\right)^n = A$

$\therefore P = \frac{A}{\left(1 + \frac{r}{100}\right)^n}$

4. $A = 2(lh + lb + bh)$

$\therefore 2(lh + lb + bh) = A$

$\therefore lh + lb + bh = \frac{A}{2}$

$\therefore lb + bh = \frac{A}{2} - lh$

$\therefore b(l + h) = \frac{A}{2} - lh$

$\therefore b = \frac{\frac{A}{2} - lh}{(l + h)}$

5. $V = s^3$

$\therefore s^3 = V$

$\therefore s = \sqrt[3]{V}$

6. $A = \frac{1}{2}h(b_1 + b_2)$

$\therefore \frac{1}{2}h(b_1 + b_2) = A$

$\therefore h(b_1 + b_2) = 2A$

$\therefore b_1 + b_2 = \frac{2A}{h}$

$\therefore b_1 = \frac{2A}{h} - b_2$

7. $S = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \frac{n}{2}[2a + (n - 1)d] = S$

$\therefore n[2a + (n - 1)d] = 2S$

$\therefore 2a + (n - 1)d = \frac{2S}{n}$

$\therefore 2a = \frac{2S}{n} - (n - 1)d$

$\therefore a = \frac{S}{n} - \frac{(n - 1)d}{2}$

Exercise 1.8

1. a) $y - x = 1$ ①
$x + y = -35$ ②
① + ②:
$2y = -34$
$\therefore y = -17$ ③

Substitute ③ into ②:
$x - 17 = -35$
$\therefore x = -18$ and $y = -17$

b) $y - x = 5$ ①
$y - 2x = 0$ ②
① − ②:
$-x - (-2x) = 5 - 0$
$\therefore x = 5$ ③

Substitute ③ into ①:
$y - 5 = 5$
$\therefore y = 10$
$\therefore x = 5$ and $y = 10$

c) $3x + 5y = 11$ ①
$4x - 3y = 34$ ②
① × 3:
$9x + 15y = 33$ ③
② × 5:
$20x - 15y = 170$ ④
③ + ④:
$29x = 203$
$\therefore x = 7$ ⑤

Substitute ⑤ into ①:
$3(7) + 5y = 11$
$\therefore 5y = -10$
$\therefore x = 7$ and $y = -2$

2. a) $x + y = 3$ ①
$y = x + 1$ ②

Substitute ② into ①:
$x + (x + 1) = 3$
$\therefore 2x = 2$
$\therefore x = 1$ ③

Substitute ③ into ②:
$y = 1 + 1 = 2$
$\therefore x = 1$ and $y = 2$

b) $x = y + 3$ ①
$x + y = 13$ ②

Substitute ① into ②:
$(y + 3) + y = 13$
$\therefore 2y = 10$
$\therefore y = 5$ ③

Substitute ③ into ①:
$x = 5 + 3 = 8$
$\therefore x = 8$ and $y = 5$

c) $x = -y$ ①
$x - y = 10$ ②
Substitute ① into ②:
$-y - y = 10$
$\therefore -2y = 10$
$\therefore y = -5$ ③

Substitute ③ into ①:
$\therefore x = 5$
$\therefore x = 5$ and $y = -5$

3. a) $x = y - 2$ ①
$x + y = 8$ ②

Substitute ① into ②:
$(y - 2) + y = 8$
$\therefore 2y = 10$
$\therefore y = 5$ ③

Substitute ③ into ①:
$x = 5 - 2 = 3$
$\therefore x = 3$ and $y = 5$

b) $y = 1 - x$ ①
$y = -2x + \frac{7}{4}$ ②

Substitute ① into ②:
$1 - x = -2x + \frac{7}{4}$
$\therefore -x + 2x = \frac{7}{4} - 1$
$\therefore x = \frac{3}{4}$ ③

Substitute ③ into ①:
$\therefore y = 1 - \frac{3}{4} = \frac{1}{4}$
$\therefore x = \frac{3}{4}$ and $y = \frac{1}{4}$

c) $x + y = 0.6$ ①
$y = -4x$ ②

Substitute ② into ①:
$x - 4x = 0.6$
$\therefore -3x = 0.6$
$\therefore x = -0.2$ ③

Substitute ③ into ②:
$\therefore y = -4(-0.2) = 0.8$
$\therefore x = -0.2$ and $y = 0.8$

d) $y = \frac{5}{4}x$ ①
$x + y = 1$ ②

Substitute ① into ②:
$\therefore x + \frac{5}{4}x = 1$
$\therefore 4x + 5x = 4$
$\therefore 9x = 4$
$\therefore x = \frac{4}{9}$ ③

Substitute ③ into ①:
$y = \frac{5}{4} \times \frac{4}{9} = \frac{5}{9}$
$\therefore x = \frac{4}{9}$ and $y = \frac{5}{9}$

e) $y = 1 - x$ ①

$y = 3x + \frac{1}{5}$ ②

Substitute ① into ②:

$1 - x = 3x + \frac{1}{5}$

$\therefore 5 - 5x = 15x + 1$

$\therefore -5x - 15x = 1 - 5$

$\therefore -20x = -4$

$\therefore x = \frac{-4}{-20} = \frac{1}{5}$ ③

Substitute ③ into ①:

$y = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore x = \frac{1}{5}$ and $y = \frac{4}{5}$

f) $x + y = -0.125$ ①

$y = -1.5x$ ②

Substitute ② into ①:

$x - 1.5x = -0.125$

$\therefore x = \frac{-0.125}{-0.5} = 0.25$ ③

Substitute ③ into ②:

$\therefore y = -1.5(0.25) = -0.375$

$\therefore x = 0.25$ and $y = -0.375$

Exercise 1.9

1. a) $\sin 33° = 0.54$

b) $\cos 15.6° = 0.96$

c) $\tan 88.2° = 31.82$

d) $\sin 67.3° = 0.92$

2. a) $\cos \theta = 0.93$

$\therefore \theta = \cos^{-1} 0.93$

$= 21.6°$

b) $\tan \theta = 1.6$

$\therefore \theta = \tan^{-1} 1.6$

$= 58.0°$

c) $\sin \theta = 0.69$

$\therefore \theta = \sin^{-1} 0.69$

$= 43.6°$

d) $\cos \theta = 0.27$

$\therefore \theta = \cos^{-1} 0.27$

$= 74.3°$

3. a) $\frac{MN}{70} = \tan 35.5°$

$\therefore MN = 70 \times \tan 35.5°$

$\therefore MN = 50$ m

(correct to the nearest metre)

b) KN = 70 m – 25 m

$= 45$ m

$\tan \theta = \frac{50}{45}$

$\therefore \theta = \tan^{-1} \frac{50}{45}$

$= 48°$

(correct to the nearest degree)

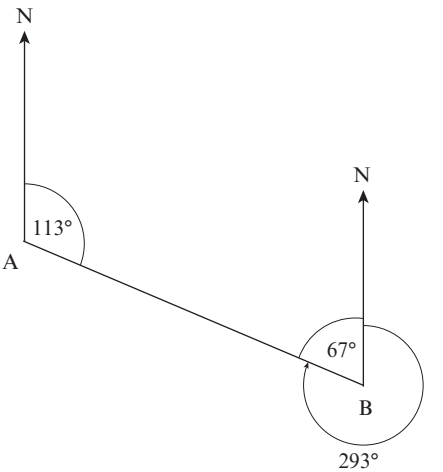
c) $\sin\theta = \frac{MN}{KM}$

$\therefore KM = \frac{MN}{\sin\theta}$

$= \frac{50}{\sin 48°}$

$= 67$ m (correct to the nearest metre)

4. Acute $A\hat{B}N = 180° - 113°$
$= 67°$
(cointerior angles)
$\therefore$ reflex $A\hat{B}N = 360° - 67°$
$= 293°$
So the bearing of point A from point B is 293°.



Exercise 1.10

1. $C = 2\pi r$
$= 2 \times \pi \times 15$
$= 94.25$ cm

2. $A = \pi r^2$
$= \pi \times 15^2$
$= 706.86$ cm^2

3. $C = 2\pi r \times \frac{\theta}{360}$
$= 2 \times \pi \times 15 \times \frac{75}{360}$
$= 19.64$ cm

4. $A = \pi r^2 \times \frac{\theta}{360}$
$= \pi \times 15^2 \times \frac{75}{360}$
$= 147.26$ cm^2

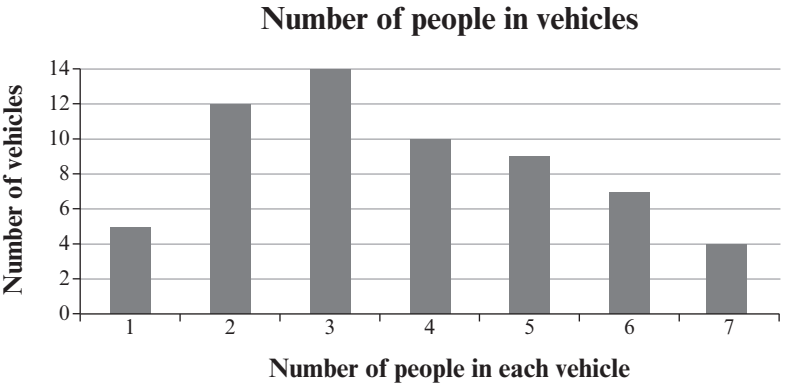
Exercise 1.11

1. a) (i) 14 **(ii)** 𝍸 𝍸 **(iii)** 7
(iv) |||| **(v)** 61

b) 61 vehicles are represented in the frequency table.

c) $(1 \times 5) + (2 \times 12) + (3 \times 14) + (4 \times 10) + (5 \times 9) + (6 \times 7) + (7 \times 4) = 226$
So 226 people are represented in the frequency table.

d)



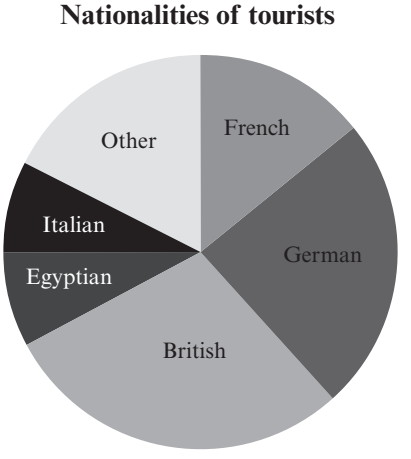
2. a) Students check the calculations of the percentages.

b) Students check the calculations of the sector angle for the French tourists.

c) German: 25% × 360° = 90°
British: 28% × 360° = 100°
Egyptian: 8% × 360° = 30°
Italian: 7% × 360° = 25°
Other: 18% × 360° = 65°

d) Students calculate the total for all the angles. They should get a total of 360°.

e)



Exercise 1.12

1. a) The ordered data set is: 2, 3, 3, 3, 4, 4, 5, 5, 7

(i) Mean = $\frac{2+3+3+3+4+4+5+5+7}{9} = 4$

(ii) There are 9 data values in the data set. The middle value is the 5th value, which is 4. So the median is 4.

(iii) Mode = 3.

(iv) Range = 7 – 2 = 5.

b) The ordered data set is: 2, 2, 4, 11, 12, 13, 13, 25, 26, 27

(i) Mean = $\frac{2+2+4+11+12+13+13+25+26+27}{10} = 13.5$

(ii) There are 10 data values in the data set. The middle value lies halfway between the 5th and the 6th data values, which are 12 and 13. $\frac{12+13}{2} = 12.5$. So the median is 12.5.

(iii) There are two modes: 2 and 13.

(iv) Range = 27 – 2 = 25.

c) The ordered data set is: –40, –34, –32, –30, –23, –21, –14, 9, 20, 28, 38

(i) Mean = $\frac{-40-34-32-30-23-21-14+9+20+28+38}{11} = -9$

(ii) There are 11 data values in the data set. The middle value is the 6th value, which is –21. So the median is –21.

(iii) There is no mode.

(iv) Range = 38 – (–40) = 78.

2. There are five data values in the data set.
The median is 9, so the 3rd data value is 9.
Mode = 6, so the 1st and 2nd data values are 6.
Range = 12, so the 5th data value is 6 + 12 = 18.
Mean = 11, so the sum of all the data values is 11 × 5 = 55.
55 – (6 + 6 + 9 + 18) = 16. So the 4th data value is 16.
The data values are therefore 6, 6, 9, 16 and 18.

TOPIC 2

Number base systems

Introduction

Your students have already been exposed to the idea that there are number bases other than base 10. For some years now, they have worked with the binary (base 2) number system. In this topic they will build on this knowledge as they work with other number bases as well (base 4, base 5, base 8 and base 16).

We start by revising place value in the decimal (base 10) system. We then extend this concept to other bases (base 4, base 5, base 8 and base 16), as your students convert whole numbers between these bases and base 10.

Then we revise the binary (base 2) system and show how to convert bicimals (base 2 fractions) to decimal fractions. Your students then convert fractions from bases 4, 5, 8 and 16 to base 10.

We move onto converting numbers between different bases by first converting the numbers to base 10, and then converting the base 10 numbers to the required base.

Then your students perform the four basic arithmetic operations (addition, subtraction, multiplication and division) on numbers in bases 4, 5, 8 and 16.

Finally your students learn how the binary, octal and hexadecimal number bases apply to computer programming. They use this knowledge to translate octal and hexadecimal data into alphabetic text, using the relevant section of the ASCII code table.

Common difficulties

When doing calculations in bases other than base 10, students can easily become confused. The procedures are exactly the same as for base 10 calculations, but students must keep in mind what number base they are working in. It can help them to write the number of the base (for example, 5) in the margin alongside the calculation and to circle it. In this way they will be able to avoid getting confused. Checking their calculations by doing the equivalent calculation in base 10 will also help them by confirming that their answer is correct, or by alerting them to a problem within the calculation.

Preparation

Make a large chart like the one below that shows all the base 2, base 4, base 5, base 8 and base 16 equivalents of the decimal numbers from 0 to 16. Display the chart in your classroom for your students to refer to when needed.

Base 10	Base 2	Base 4	Base 5	Base 8	Base 16
0	0	0	0	0	0
1	1	1	1	1	1
2	10	2	2	2	2
3	11	3	3	3	3
4	100	10	4	4	4
5	101	11	10	5	5
6	110	12	11	6	6
7	111	13	12	7	7
8	1 000	20	13	10	8
9	1 001	21	14	11	9
10	1 010	22	20	12	A
11	1 011	23	21	13	B
12	1 100	30	22	14	C
13	1 101	31	23	15	D
14	1 110	32	24	16	E
15	1 111	33	30	17	F
16	10 000	40	31	20	10

Diennes blocks are a useful visual aid that you can use to illustrate place value in the base 10 system.

Introduction for students

Write some binary numbers on the board without explaining what they are. Ask your students whether or not they recognise these numbers from previous years. See if any of them classify these numbers as binary numbers.

Now ask your class if they remember how to convert a binary number to a decimal number. Write a six-digit binary number on the board, for example 110 101 and see if your students are able to convert it to its decimal equivalent, which is 53. If necessary, revise the method for doing this:

$$\begin{aligned} 110\ 101 &= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 53 \end{aligned}$$

Explain to your class that they are now going to learn about other number bases as well.

Answers

Exercise 2.1

1. a) $1 \times 10^2 + 6 \times 10^1 + 9 \times 10^0$
b) $9 \times 10^3 + 2 \times 10^1 + 8 \times 10^0$
c) $2 \times 10^4 + 8 \times 10^3 + 3 \times 10^0$
d) $4 \times 10^5 + 5 \times 10^3 + 2 \times 10^2 + 9 \times 10^1 + 1 \times 10^0$

2. a) 6 473 **b)** 43 210 **c)** 926 085 **d)** 2 070 800

Exercise 2.2

1. a) $2 \times 4^1 + 3 \times 4^0 = 11$
b) $1 \times 4^2 + 2 \times 4^1 = 24$
c) $2 \times 4^3 + 1 \times 4^2 + 3 \times 4^0 = 147$
d) $3 \times 4^4 + 1 \times 4^3 + 2 \times 4^2 + 1 \times 4^1 + 1 \times 4^0 = 869$

2. a) 33_4 **b)** $1\ 000_4$ **c)** $12\ 030_4$ **d)** $223\ 131_4$

Exercise 2.3

1. a) 22 **b)** 40 **c)** 117 **d)** 403

2. a) 100_5 **b)** 220_5 **c)** $3\ 432_5$ **d)** $242\ 103_5$

Exercise 2.4

1. a) 51 **b)** 141 **c)** 508 **d)** 1 904

2. a) 100_8 **b)** 275_8 **c)** $7\ 332_8$ **d)** $31\ 267_8$

Exercise 2.5

1. a) 57 **b)** 678 **c)** 3 470 **d)** 48 132

2. a) 11_{16} **b)** 211_{16} **c)** $115C_{16}$ **d)** $2D6A_{16}$

Exercise 2.6

1. 7 **2.** 14 **3.** 23 **4.** 45
5. 49 **6.** 74 **7.** 223 **8.** 627

Exercise 2.7

1. 0.5 **2.** 1.25 **3.** 2.375 **4.** 3.6875
5. 5.03125 **6.** 7.34375 **7.** 10.40625 **8.** 11.796875

Exercise 2.8

1. 3.5 **2.** 6.1875 **3.** 4.44 **4.** 16.864
5. 5.9219 **6.** 20.1016 **7.** 9.9063 **8.** 28.5391

Exercise 2.9

1. a) $12\ 243_5$ **b)** $1\ 664_8$ **c)** $3B4_{16}$

2. a) $10\ 033_4$ **b)** 417_8 **c)** $10F_{16}$

3. a) $233\ 100_4$ **b)** $44\ 044_5$ **c)** $BD0_{16}$

4. a) $303\ 220_4$ **b)** $101\ 204_5$ **c)** $6\ 350_8$

Exercise 2.10

1. a) 332_4 **b)** $1\ 001_4$ **c)** 444_5 **d)** $1\ 422_5$
e) 675_8 **f)** $2\ 143_8$ **g)** $69E_{16}$ **h)** $8F2_{16}$

2. a) 33 + 29 = 62 **b)** 47 + 18 = 65
c) 69 + 55 = 124 **d)** 148 + 89 = 237
e) 377 + 68 = 445 **f)** 645 + 478 = 1 123
g) 637 + 1 057 = 1 694 **h)** 940 + 1 350 = 2 290

Exercise 2.11

1. a) 201_4 **b)** 122_4 **c)** $1\ 110_5$ **d)** $1\ 141_5$
e) 440_8 **f)** $4\ 623_8$ **g)** 11_{16} **h)** 217_{16}

2. a) 63 – 30 = 33
b) 53 – 27 = 26
c) 358 – 203 = 155
d) 486 – 315 = 171
e) 500 – 212 = 288
f) 2 631 – 180 = 2 451
g) 489 – 472 = 17
h) 3 107 – 2 572 = 535

Exercise 2.12

1. a) $13\,020_4$ **b)** $23\,230_4$ **c)** $3\,023_5$ **d)** $23\,131_5$
e) $1\,174_8$ **f)** $20\,733_8$ **g)** $2\,4C6_{16}$ **h)** $3DBF_{16}$

2. a) 57 × 8 = 456
b) 44 × 17 = 748
c) 97 × 4 = 388
d) 238 × 7 = 1 666
e) 53 × 12 = 636
f) 321 × 27 = 8 667
g) 523 × 18 = 9 414
h) 479 × 33 = 15 807

Exercise 2.13

1. a) 21_4 **b)** 11_4 **c)** 44_5 **d)** 41_5
e) 41_8 **f)** 13_8 **g)** 23_{16} **h)** $1D_{16}$

2. a) 27 ÷ 3 = 9
b) 45 ÷ 9 = 5
c) 96 ÷ 4 = 24
d) 651 ÷ 31 = 21
e) 528 ÷ 16 = 33
f) 165 ÷ 15 = 11
g) 140 ÷ 4 = 35
h) 1 421 ÷ 49 = 29

Exercise 2.14

1. a) $64\,317_8$ **b)** $156\,021_8$ **c)** $123\,173_8$

2. a) $68CF_{16}$ **b)** $DC11_{16}$ **c)** $A67B_{16}$

3. Student's own answers, for example: Octal and hexadecimal data is much shorter than binary data, so it is quicker to read and write. Strings of zeros and ones are also harder to make sense of than digits or letters.

Exercise 2.15

1. HARDWARE
2. SOFTWARE
3. DATA
4. KEYBOARD
5. SCREEN; PRINTER

Assess your progress

1. 3 612 095

2. **a)** 483 **b)** 358 **c)** 966 **d)** 2 522

3. **a)** $31\,041_4$ **b)** $11\,412_5$ **c)** $1\,531_8$ **d)** 359_{16}

4. **a)** 6.625 **b)** 5.875 **c)** 11.76
d) 11.5781 **e)** 59.3633

5. **a)** 261_8 **b)** $12\,212_4$ **c)** $83C_{16}$ **d)** $41\,133_5$

6. **a)** $3\,010_5$ **b)** $93A_{16}$ **c)** 111_4 **d)** 673_8
e) $11\,022_4$ **f)** $3\,516_{16}$ **g)** 102_5 **h)** 32_8

7. **a)** bit **b)** byte **c)** ASCII **d)** hardware
e) software **f)** data **g)** keyboard **h)** screen

TOPIC 3

Modular arithmetic

Introduction

In this topic, your students are introduced to modular arithmetic for the first time.

First we revise the terminology that relates to division (dividend, divisor, quotient and remainder) and we explain that modular arithmetic is based on the concept of the remainder of a division. We introduce the concept of a cyclic event and relate this to situations in everyday life. The examples that we use include the following:

- The days of the week start on Sunday (for example) and rotate through Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and back to Sunday.
- The hour hand of an analogue clock moves from 1 all the way round to 12, and then back again to 1.
- The days of the year rotate from 1 through to 365 (except for leap years) and then back again to 1.
- The months of the year rotate from January through to December and then back again to January.
- The angles on the Cartesian plane are measured in an anti-clockwise direction from 0º through 90º, 180º and 270º to 360º, which corresponds with 0º. Your students will learn more about this in the trigonometry topic.

Next, your students work with residues and remainders when simplifying positive integers in terms of a given modulus.

These concepts are extended to dealing with negative integers as well.

Then your students learn how to add, subtract and multiply positive and negative integers in modular arithmetic. They also investigate some properties of modular arithmetic and test further conjectures for themselves.

Finally your students use modular arithmetic to solve problems in daily life.

Common difficulties

Students sometimes find the idea of a 'new' arithmetic confusing and daunting. Reassure them that doing calculations in modular arithmetic is in fact far easier than the calculations that they did in the previous topic about number bases. Keep reminding them that they are working with remainders of integer divisions in different moduli.

Preparation

Find visual aids that depict cyclic events and display them around your classroom. Obvious choices are calendars and clock faces. You could also display pictures that depict the seasons (spring, summer, autumn and winter), phases of the moon, timetables for shift duty, or talk about menstrual cycles and any other cyclic events from real life that your students will relate to.

Prepare some charts that show examples of residues and basic arithmetic in different modulos, from mod 2 up to mod 12.

Introduction for students

Write the following division problems on the board.

$1 \div 5 = 0$ rem 1
$2 \div 5 = 0$ rem 2
$3 \div 5 = 0$ rem 3
$4 \div 5 = 0$ rem 4
$5 \div 5 = 1$ rem 0
$6 \div 5 = 1$ rem 1
$7 \div 5 = 1$ rem 2
$8 \div 5 = 1$ rem 3
$9 \div 5 = 1$ rem 4
$10 \div 5 = 2$ rem 0
$11 \div 5 = 2$ rem 1
$12 \div 5 = 2$ rem 2
$13 \div 5 = 2$ rem 3
$14 \div 5 = 2$ rem 4
$15 \div 5 = 3$ rem 0

Ask your class to look for a pattern in the remainders. Ask a couple of them to describe this pattern in their own words. They should see that the remainders all have values of 0, 1, 2, 3 or 4, and that these numbers repeat in a fixed pattern.

Explain to your class that this concept forms the basis of a new kind of arithmetic that they will learn about in this topic.

Answers

Exercise 3.1

1. a) 0 and 1
b) 0, 1, 2 and 3
c) 0, 1, 2, 3 and 4
d) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11

2. a) $3 = 1 \bmod 2$
b) $9 = 2 \bmod 7$
c) $12 = 4 \bmod 8$
d) $15 = 0 \bmod 5$
e) $16 = 1 \bmod 3$
f) $19 = 3 \bmod 4$
g) $49 = 1 \bmod 3$
h) $34 = 4 \bmod 6$
i) $20 = 6 \bmod 7$
j) $29 = 2 \bmod 9$
k) $34 = 10 \bmod 12$
l) $58 = 8 \bmod 10$

Exercise 3.2

1. $-3 = 1 \bmod 2$
2. $-9 = 5 \bmod 7$
3. $-12 = 4 \bmod 8$
4. $-15 = 0 \bmod 5$
5. $-16 = 2 \bmod 3$
6. $-19 = 1 \bmod 4$
7. $-49 = 2 \bmod 3$
8. $-34 = 2 \bmod 6$
9. $-20 = 1 \bmod 7$
10. $-29 = 7 \bmod 9$
11. $-34 = 2 \bmod 12$
12. $-58 = 2 \bmod 10$

Exercise 3.3

1. a) $1 \bmod 3$
b) $5 \bmod 8$
c) $0 \bmod 5$
d) $4 \bmod 6$
e) $7 \bmod 9$
f) $2 \bmod 12$
g) $6 \bmod 10$
h) $3 \bmod 4$
i) $3 \bmod 7$
j) $10 \bmod 11$

2. a) $1 \bmod 3$
b) $5 \bmod 8$
c) $0 \bmod 5$
d) $4 \bmod 6$
e) $7 \bmod 9$
f) $2 \bmod 12$
g) $6 \bmod 10$
h) $3 \bmod 4$
i) $3 \bmod 7$
j) $10 \bmod 11$

3. a) The corresponding answers are the same.
b) Yes, because the order in which you add the numbers makes no difference to the answer.

Exercise 3.4

1. a) 6 mod 10 **b)** 1 mod 3
c) 3 mod 4 **d)** 0 mod 8
e) 2 mod 11 **f)** 5 mod 7
g) 5 mod 6 **h)** 4 mod 5
i) 4 mod 9 **j)** 9 mod 12

2. a) 4 mod 10 **b)** 2 mod 3
c) 1 mod 4 **d)** 0 mod 8
e) 9 mod 11 **f)** 2 mod 7
g) 1 mod 6 **h)** 1 mod 5
i) 5 mod 9 **j)** 3 mod 12

3. Look carefully at your answers to Questions 1 and 2.
a) The corresponding answers are not the same.
b) No, because the order in which you subtract the numbers makes a big difference to the answer.

Exercise 3.5

1. a) 3 mod 4 **b)** 5 mod 7
c) 4 mod 5 **d)** 6 mod 9
e) 3 mod 6 **f)** 9 mod 12
g) 1 mod 3 **h)** 8 mod 10
i) 3 mod 11 **j)** 4 mod 8

2. a) 3 mod 4 **b)** 5 mod 7
c) 4 mod 5 **d)** 6 mod 9
e) 3 mod 6 **f)** 9 mod 12
g) 1 mod 3 **h)** 8 mod 10
i) 3 mod 11 **j)** 4 mod 8

3. a) The corresponding answers are the same.
b) Yes, because the order in which you multiply the numbers makes no difference to the answer.

Exercise 3.6

1. a) (i) $[(25 \bmod 7) \times (34 \bmod 7)] \bmod 7$
$= (4 \times 6) \bmod 7 = 24 \bmod 7 = 3$
(ii) $(25 \times 34) \bmod 7 = 850 \bmod 7 = 3$
b) (i) $[(58 \bmod 5) \times (17 \bmod 5)] \bmod 5$
$= (3 \times 2) \bmod 6 = 6 \bmod 5 = 1$
(ii) $(58 \times 17) \bmod 5 = 986 \bmod 5 = 1$
c) $[(a \bmod n) \times (b \bmod n)] \bmod n = ab \bmod n$

2. a) [17 mod 5 + (−17 mod 5)] mod 5
= (2 + 3) mod 5 = 5 mod 5 = 0
b) [51 mod 9 + (−51 mod 9)] mod 9
= (6 + 3) mod 9 = 9 mod 9 = 0
c) [(a mod n) + (−a mod n)] = 0

3. a) False: students provide their own counter-examples.
b) True
c) False: students provide their own counter-examples.
d) True

Exercise 3.7

1. a) 1 m **b)** 2 m
c) 1 m **d)** 2 m

2. 92 = 8 mod 12. 8 months after March is November, so her investment will mature in November.

3. Pizza is measured in mod 7 and hot dogs in mod 9.
189 = 0 mod 7 and 189 = 0 mod 9. So both meals will be on the menu in 189 days' time.

4. Pekun's 21st birthday will be in the year 2014 + 21 = 2035. There are five leap years in that time (2016, 2020, 2024, 2028 and 2032). So the number of days till his 21st birthday are 365 × 21 + 5 = 7 670.
7 670 = 5 mod 7 and 5 days after Friday is Wednesday.
So Pekun's 21st birthday will fall on a Wednesday.

5. a) (i) 5 + 6 + 8 + 5 + 1 + 5 + 1 + 1 + 1 + 2 + 2 + 1 = 38
38 = 8 mod 10, so the last digit will be a 1.
(ii) 3 + 7 + 1 + 2 + 9 + 8 + 2 + 2 + 6 + 6 + 4 + 7 = 57
57 = 7 mod 10, so the last digit will be a 2.
(iii) 3 + 3 + 4 + 1 + 5 + 3 + 3 + 1 + 2 + 5 + 1 + 8 = 39
39 = 9 mod 10, so the last digit will be a 0.
(iv) 4 + 9 + 1 + 3 + 8 + 2 + 8 + 2 + 8 + 3 + 6 + 1 = 55
55 = 5 mod 10, so the last digit will be a 4.
b) If one of the digits is entered incorrectly, the sum of all 13 digits will be anything from 9 less to 9 more. So the sum of the digits will no longer be 9 (mod 10). For the sum to remain 9 (mod 10), the incorrect digit would have to be 10 more or 10 less than the correct digit, and this is not possible.

Assess your progress

1. **a)** 0, 1, 2, 3, 4 and 5
b) 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9

2. **a)** $20 = 2 \bmod 3$ **b)** $36 = 1 \bmod 5$
c) $43 = 3 \bmod 8$ **d)** $79 = 7 \bmod 12$
e) $-20 = 1 \bmod 3$ **f)** $-36 = 4 \bmod 5$
g) $-43 = 5 \bmod 8$ **h)** $-79 = 5 \bmod 12$

3. **a)** $1 \bmod 3$ **b)** $2 \bmod 4$
c) $1 \bmod 6$ **d)** $9 \bmod 11$
e) $2 \bmod 9$ **f)** $4 \bmod 5$
g) $7 \bmod 10$ **h)** $8 \bmod 9$
i) $1 \bmod 2$ **j)** $3 \bmod 5$
k) $4 \bmod 11$ **l)** $5 \bmod 7$
m) $3 \bmod 4$ **n)** $2 \bmod 6$
o) $4 \bmod 5$ **p)** $5 \bmod 8$
q) $3 \bmod 7$ **r)** $9 \bmod 12$

4. **a)** $(11 + 3 + 8) \bmod 12 = 10$ and $11 \oplus 3 \oplus 8 \pmod{12} = 10$: true
b) $(16 + 4 + 5) \bmod 6 = 1$ and $16 \bmod 6 + 4 \bmod 6 + 5 \bmod 6 = 4 + 4 + 5 = 13$: false
c) $(6 \times 7 \times 5) \bmod 3 = 0$ and $6 \otimes 7 \otimes 5 \pmod 3 = 0$: true
d) $[(32 \bmod 7) + (-32 \bmod 7)] = 0$: true
e) $65 \bmod 8 - 61 \bmod 5 = 0$ and $(65 - 61) \bmod (8 - 5) = 1$: false
f) $35 \bmod 11 + 21 \bmod 8 = 7$ and $20 \bmod 13 = 7$: true
g) $[(8 \bmod 10) \times (67 \bmod 10)] \bmod 10 = 6$ and $(8 \times 67) \bmod 10 = 6$: true
h) $(2 \times 11 \times 15) \bmod 4 = 2$ and $2 \bmod 4 \times 11 \bmod 4 \times 15 \bmod 4 = 18$: false

5. **a)** 7 **b)** 1

6. 10 weeks = 70 days. $70 = 0 \bmod 14$ (for every two weeks) and $70 = 0 \bmod 5$, so Omobola will have both classes again in exactly 10 weeks' time.

TOPIC 4

Indices and standard form

Introduction

Most of the work in this topic has been covered in previous grades. Students should already have a solid foundation on which to build new concepts and learn to solve more complex problems.

First we revise approximation which includes rounding off numbers to the nearest 10, 100 or 1 000 to give an appropriate estimate. We also revise the rules for rounding off to a given number of decimal places (d.p.) or significant figures (s.f.).

Then we study the method of writing very large or very small numbers in standard form:

$A \times 10^n$ where $1 \leq A < 10$ and n is a positive or negative integer

We learn how to convert numbers from standard form to decimal notation and from decimal notation into standard form.

We use the four mathematical operations, addition, subtraction, multiplication and division, to calculate with numbers in standard form. You can use the following guidelines to explain this to students:

- When multiplying or dividing numbers that are written in standard form, multiply (or divide) the number part and the powers of ten parts separately. Write the final answer in correct standard form.
- You may only add or subtract numbers written in standard form when the powers of ten are the same.
- First rewrite numbers where necessary, making sure that the powers of ten are the same before adding or subtracting. Write the final answer in correct standard form.
- Writing numbers in standard form can make calculations simpler. Calculators can not always perform calculations on numbers that are too large or too small. The standard form method of writing numbers can save time and avoid unnecessary errors that may occur when working with numbers with many decimal places.
- Notice that a negative index indicates a small number and a positive index indicates a large number.

The laws of indices are revised. A chart of the laws is provided below.

Then introduce students to fractional indices. A root can be written as a fraction index.

Fractional indices follow the same rules as integral indices. Keep referring to the laws to explain how they are used in simplifying expressions or solving equations.

Then solve equations that contain indices. Indical equations have indices that are variables.

We look at situations in real life that are examples of exponential growth or decay and see how to solve practical problems.

Common difficulties

When changing numbers into standard form or expanded form, students often move the decimal place in the wrong direction.

Remember when you convert from standard form to decimal notation (or expanded form), the index, n, in the formula $A \times 10^n$ tells you how many places to move the decimal point:

- For $n > 0$, move the decimal point to the right.
- For $n < 0$, move the decimal point to the left.

Check that the students are applying the index laws correctly. Some students multiply the bases of powers together. Remind students to keep the base the same and only add the indices when multiplying terms that contain indices.

When solving indical equations, ask the students to check which type of problem they have. Look out for the more complicated indical equations which involve taking out common factors or factorising trinomials.

Preparation

Enlarge the chart below for the classroom wall.

Vocabulary
Indices is the plural of index.
Exponent is another word used for index.
The word **power** is also sometimes used instead of exponent or index.

Standard form
$A \times 10^n$ where $1 \leq A < 10$ and n is a positive or negative integer.

Powers of ten

10 000	$10 \times 10 \times 10 \times 10$	10^4
1 000	$10 \times 10 \times 10$	10^3
100	10×10	10^2
10	10	10^1
1	1	10^0
0.1	$\frac{1}{10}$	10^{-1}
0.01	$\frac{1}{100}$	10^{-2}
0.001	$\frac{1}{1\,000}$	10^{-3}

The laws of indices

$a^m \times a^n = a^{m+n}$	Addition law
$a^m \div a^n = a^{m-n}$	Subtraction law
$(a^m)^n = a^{mn}$	Product law
$a^0 = 1$	Any base with index 0 is equal to 1
$a^{-n} = \frac{1}{a^n}$	A negative index inverts the base

Introduction for students

Write some very large and very small numbers on the board and ask the students how they can round them off in different ways. Then introduce the standard notation $A \times 10^n$ and discuss how to write the given numbers in standard notation.

Introduce each section by working through the worked examples before asking students to complete the given Exercises.

Answers

Exercise 4.1

1. **a)** 4 800 **b)** 55 000 **c)** 11 100
d) 67 000 **e)** 79 000 **f)** 100 000

2. **a)** 5 160 **b)** 50 210 **c)** 10 200
d) 9 990 **e)** 8 030 **f)** 560

3. **a)** 15.0 **b)** 2.6 **c)** 101.1
d) 78.3 **e)** 65.5 **f)** 5.6

4. **a)** 261.06 **b)** 12.50 **c)** 101.01
d) 65.05 **e)** 5.34 **f)** 15.51

5. **a)** 6 000 **b)** 0.09 **c)** 7 000
d) 0.003 **e)** 1 **f)** 20

6. **a)** 11 000 **b)** 6 100 **c)** 37 000
d) 0.0029 **e)** 0.08 **f)** 31

Exercise 4.2

1. **a)** 312.6 **b)** 0.05001 **c)** 940 700
d) 0.000678 **e)** 4 103.02 **f)** 2 424 000
g) 0.007986 **h)** 17 831 000

2. 1 000 000 000 000

3. 5 895 m

Exercise 4.3

1. **a)** 6.401×10^{4} **b)** 8.1×10^{-3} **c)** 1.25×10^{5}
d) 2.0403×10^{6} **e)** 2.56×10^{-4} **f)** 6.25×10^{8}
g) 1.009×10^{-3} **h)** 1.18024×10^{2} **i)** 1.090807×10^{3}

2. 1.785×10^{8}

3. 3.793×10^{7}

Exercise 4.4

1. a) 3×10^7 **b)** 8.05×10^8 **c)** 1.26×10^2
d) 2.32×10^2 **e)** 3.2032×10^{-5} **f)** 1.96×10^6

2. a) 1.5×10^6 **b)** 3.54×10^2 **c)** 4.4×10^4
d) 9×10^{10} **e)** 2×10^{-3} **f)** 3.7×10^4

3. a) 9.18×10^5 **b)** 918 000 **c)** 920 000

4. a) 4.47×10^3 **b)** 4 470 **c)** 4 500

Exercise 4.5

1. a) 8.024×10^4 **b)** 6.408×10^5 **c)** 9.5936×10^{-2}
d) 5.171×10^{-3} **e)** 9.22047×10^{-2} **f)** 9.448×10^5

2. a) 7.1465×10^5 **b)** 714 650 **c)** 715 000

3. a) 7.55433×10^{-2} **b)** 0.0755433 **c)** 0.08
d) 0.0755

Exercise 4.6

1. 1 000 000 000 000 000
2. 1.496×10^8 m
3. 342 m/s
4. 1.08×10^{15} m/h or 1 080 000 000 000 000 m/h
5. Time = $7.78 \times \frac{108}{3} \times 105 \times \frac{1}{60}$ = 43 minutes
6. 2.503×10^{13}

Exercise 4.7

1. a) $9^5 \times 9^7 = 9^{12}$ **b)** $8^6 \times 8^{10} = 8^{16}$ **c)** $2^8 \times 2^{10} = 2^{18}$
d) $4^x \times 4^x = 4^{2x}$ **e)** $2^{3^x} \times 2^{4^x} = 2^{7^x}$

2. a) $v^7 \times v^8 = v^{15}$ **b)** $p^3 \times p^5 = p^8$ **c)** $q^6 \times q^{11} = q^{17}$

3. a) $\frac{x12}{x8} = \frac{x^3 \times x^4}{x^3} = x^4$

b) $\frac{v^7 \times v^9}{v^6} = \frac{v^7 \times v^6 \times v^3}{v^6} = v^7 \times v^3 = v^{10}$

c) $\frac{v^{12}}{v^{17}} = \frac{v^{12}}{v^{12} \times v^5} = \frac{1}{v^5}$

4. a) $4^5 + 4^6 = 4^5 + 4^5 \times 4^1 = 4^5(1 + 4) = 5(4^5)$

b) $5^3 + 5^3 = 2(5^3)$

c) $\frac{4^{x+1}}{4} = \frac{4^x \times 4^1}{4^1} = 4x$

d) $\frac{5^{x+2}}{25} = \frac{5^x \times 5^2}{5^2} = 5^x$

Exercise 4.8

1. a) $3^{20} \div 3^7 = 3^{13}$ **b)** $5^{17} \div 5^{14} = 5^3$ **c)** $8^{40} \div 8^{27} = 8^{13}$
d) $u^{16} \div u^{13} = u^3$ **e)** $r^{21} \div r^{19} = r^2$

2. a) $\frac{2^{x+5}}{2^x}$
$= 2^{x+5-x}$
$= 2^5$

b) $\frac{3^{x+7}}{3^{x+5}}$
$= 3^{x+7-(x+5)}$
$= 3^{x+7-x-5}$
$= 3^2$

c) $\frac{5^{7x+12}}{5^{7x+7}}$
$= 5^{7x+12-(7x+7)}$
$= 5^{7x+12-7x-7}$
$= 5^5$

d) $\frac{7^{12x+10}}{7^{11x+6}}$
$= 7^{12x+10-(11x+6)}$
$= 7^{12x+10-11x-6}$
$= 7^{x+4}$

3. a) $3^4 - 3^3$
$= 3^3 \times 3^1 - 3^3$
$= 3^3(3^1 - 1)$
$= 2(3^3)$

b) $7^8 - 7^7$
$= 7^7 \times 7^1 - 7^7$
$= 7^7(7^1 - 1)$
$= 6(7^7)$

c) $9^6 - 9^5$
$= 9^5(9^1 - 1)$
$= 8(9^5)$

Exercise 4.9

1. a) $10^0 = 1$ **b)** $17^0 = 1$ **c)** $(-20)^0 = 1$
d) $1\,500^0 = 1$ **e)** $3\,000^0 = 1$

2. a) $\frac{1}{3^{10}} = 3^{-10}$ **b)** $\frac{1}{5^4} = 5^{-4}$ **c)** $\frac{1}{1\,000} = \frac{1}{10^3} = 10^{-3}$

d) $\frac{1}{64} = \frac{1}{8^2} = 8^{-2}$ **e)** $\frac{1}{34^3} = 34^{-3}$

3. a) $3^{-7} = \frac{1}{3^7}$ **b)** $7^{-4} = \frac{1}{7^4}$ **c)** $11^{-4} = \frac{1}{11^4}$

4. a) $\frac{y7}{y9} = y^{-2}$ **b)** $\frac{x^5y^7}{x^7y^{12}} = x^{-2}y^{-5}$ **c)** $\frac{3x}{3^{2x}} = 3^{-x}$

d) $\frac{5^{x+1}}{5^{3x+7}}$
$= 5^{x+1-(3x+7)}$
$= 5^{x+1-3x-7}$
$= 5^{-2x-6}$

Exercise 4.10

1. 5^{21} **2.** 8^{30} **3.** 2^{18}

4. 2^4 **5.** 7^{-6} **6.** $(3a^2)^2 = 9a^4$

7. $(2x^2y)^3 = 8x^6y^3$ **8.** $(5ab^3)^2 = 25a^2b^6$ **9.** $(27a^{-3})^{-\frac{2}{3}} = \frac{a^2}{9}$

10. $(2x^{-1})^{-3} = \frac{x^3}{8}$

Exercise 4.11

1. a) $4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$ **b)** $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$

c) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^3 = 27$ **d)** $49^{\frac{3}{2}} = (7^2)^{\frac{3}{2}} = 7^3 = 343$

e) $8^{\frac{5}{3}} = (2^3)^{\frac{5}{3}} = 2^5 = 32$

2. a) $4^{-\frac{3}{2}} = (2^2)^{-\frac{3}{2}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

b) $8^{-\frac{2}{3}} = (2^2)^{-\frac{2}{3}} = 2^{-2} - \frac{1}{2^2} = \frac{1}{4}$

c) $9^{2.5} = 9^{\frac{5}{2}} = (3^2)^{\frac{5}{2}} = 3^5 = 243$

d) $25^{1.5} = 25^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = 5^3 = 125$

e) $49^{-\frac{3}{2}} = (7^2)^{-\frac{3}{2}} = 7^{-3} = \frac{1}{7^3} = \frac{1}{343}$

3. a) $\left[\left(\frac{1}{3}\right)^{-6}\right]^{-\frac{1}{2}} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

b) $\left[\left(\frac{1}{7}\right)^{-\frac{1}{3}}\right]^6 = \left(\frac{1}{7}\right)^{-2} = \left(\frac{1}{\frac{1}{7}}\right)^2 = \left(\frac{1}{\frac{1}{49}}\right) = 49$

c) $\left(\frac{8}{7}\right)^{-1} = \left(\frac{1}{\frac{8}{7}}\right)^1 = \frac{7}{8}$

4. a) $100^{\frac{3}{2}} = (10^2)^{\frac{3}{2}} = 10^3 = 1\,000$

b) $1\,000^{\frac{2}{3}} = (10^3)^{\frac{2}{3}} = 10^2 = 100$

c) $\sqrt[4]{2^8} = (2^8)^{\frac{1}{4}} = 2^2 = 4$

d) $\sqrt[5]{7^{10}} = (7^{10})^{\frac{1}{5}} = 7^2 = 49$

Or $7^{\frac{10}{5}} = 7^2 = 49$

Exercise 4.12

1. $x = \pm 3$ **2.** $x = -2$

3. no real solution **4.** $x = -4$

5. $x = \pm 3$
6. $x = 3$
7. $x = -2$
8. $x = 2$
9. $x = \pm 3$ or $x = \pm 2$
10. $x = 3$
11. $x = 16$
12. $x = \frac{81}{16}$

Exercise 4.13

1. a) $x = 3$
b) $x = 3$

2. a) $x = 0$
b) $x = 2$

3. a) $x = 2$
b) $x = 2$

4. a) $x = \frac{4}{5}$
b) $x = 1$

5. a) $x = -6$
b) $x = -4$ or $x = \frac{1}{2}$

6. a) $x = -2$
b) $x = \frac{2}{3}$

7. a) $x = -\frac{1}{8}$
b) $x = \frac{3}{5}$

8. a) $x = \pm 6$
b) $x = 2$

9. a) $x = \frac{7}{2}$
b) $x = \frac{5}{2}$

10. a) $x = -1$ or $x = \frac{3}{2}$
b) $x = -1$

Exercise 4.14

1. ₦712 266
2. ₦1 378 580
3. 11 756
4. 570
5. a) $w = 2^4 - 1 = 15$cm
b) $w = 2^0 - 1 = 0$ cm
c) $63 = 2^t - 1 \therefore t = 6$ hours

Assess your progress

1. a) (i) 10 **(ii)** 13.10 **(iii)** 13.1
b) (i) 30 **(ii)** 30.91 **(iii)** 30.9
c) (i) 210 **(ii)** 206.05 **(iii)** 206
d) (i) 1070 **(ii)** 1 070.75 **(iii)** 1 070
e) (i) 80 **(ii)** 77.96 **(iii)** 78.0
f) (i) 6 340 **(ii)** 6 341.22 **(iii)** 6 340

2. a) 407 000 **b)** 0.0003981 **c)** 23 140 000
d) 383 690 **e)** 0.0000706 **f)** 9 102 000

3. a) 8.945×10^{6} **b)** 5.67×10^{2} **c)** 5.0302×10^{5}
d) 3.65×10^{-2} **e)** 7.5×10^{6} **f)** 4.32×10^{-4}

4. a) 2.73×10^{8} **b)** 3.486×10^{9} **c)** 4.484×10^{3}
d) 1.705×10^{3}

5. a) 7.84×10^{2} **b)** 9.12×10^{10} **c)** 2.35×10^{-3}
d) 2.65×10^{14}

6. a) 9.195×10^{4} **b)** 3.65×10^{5} **c)** 8.016×10^{-2}
d) 4.985×10^{-3}

7. a) 4.85×10^{5} **b)** 485 000

8. 1.44×10^{11}

9. 235

10. a) 4^{9} **b)** 3^{-6} **c)** 2^{-2}
d) 6^{3} **e)** 7^{8} **f)** 10^{3}

11. a) $4a^{10}$ **b)** $125y$ **c)** a^{12}
d) 1 **e)** $4x$ **f)** a^{6}

12. a) 4 **b)** 4 **c)** 1
d) $\frac{1}{5}$ **e)** 8 **f)** $\frac{10}{3}$

13. a) $x = 0$ **b)** $x = 2$ **c)** $x = -5$
d) $x = \pm 12$ **e)** $x = 4$ **f)** $x = \frac{3}{2}$
g) $x = 1$ **h)** $x = \pm 32$ **i)** $x = \pm \frac{9}{7}$
j) $x = 4$ **k)** $x = 3$ **l)** $x = -2$

14. a) $x = \frac{2}{5}$ **b)** $x = 4$ **c)** $x = \frac{3}{2}$
d) $x = 0$ or 1

15. $\frac{1}{25}$

16. -1

TOPIC 5

Logarithms

Introduction

A logarithm is an index and students have already learnt all the laws of indices. They need to understand indices to be able to expand on this knowledge and understand logarithms.

We first look at the definition of a logarithm and how to change numbers from index form to logarithm form. Then we simplify logarithms by writing them in index form and applying the laws of indices.

We learn that logarithms with base 10 may be calculated using a calculator or logarithm tables, which are available at the back of the Student's Book.

We look at the graph of $y = 10^x$ and notice that it is a curve rather than a straight line.

Then we study the laws of logarithms and see how they are similar to the laws of indices.

We then apply the laws of logarithms to simplify expressions and also learn the change of base law.

We learn how to read logarithm and antilogarithm tables and use these to find answers to numerical calculations.

Finally, we apply logarithms to capital markets where logarithms are used to calculate the length of time of an investment.

Common difficulties

The same difficulties that students encounter with indices occur in this section of work. Students need to understand the definition of a logarithm very clearly. They also need to be able to change powers from index form to logarithm form or logarithm form to index form.

Remember that the base has to be a positive real number but may not equal 1. You may only take the logarithm of a positive number and the logarithm of 1 is always equal to zero. Make sure the students understand why this is the case.

Any logarithm $\log_a a = 1$ as $a^1 = a$ for any value of $a > 0$, $a \neq 1$.

The logarithm of 1 with any positive base always equals 0. $\log_m 1 = 0$, where $m > 0$ and $m \neq 1$.

Students also do not always read the logarithm tables properly. When they are finding the log of a number, they need to use the logarithm tables. When they convert back to the answer, they need to use the antilogarithm tables.

Show your students a logarithm table booklet and explain that the logarithm and antilogarithm tables for logarithms with base 10 are provided at the end of their textbooks for reference. These tables can also be found in a logarithm table booklet.

Go through one or two examples with your students so that they can see how to read the answers correctly from the table.

First find logarithms of a few two-digit numbers, then three-digit numbers so that they can see how to use the columns.

Give a few examples of logs of four-digit numbers so that students can see how to use the 'Differences' columns in the table.

Preparation

Provide a chart of the logarithm definitions and laws as well as the log graph.

Definition of a logarithm

The logarithm of a number is the power to which the base of the number is raised to give the number.

The laws of logarithms

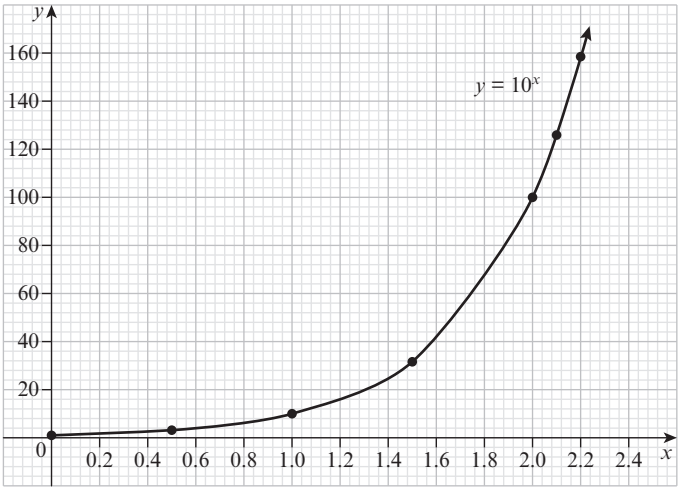
Product law $\log a + \log b = \log ab$

Quotient law $\log a - \log b = \log \frac{a}{b}$

Power law $\log ab = b \log a$

Change of base law If $\log_a b = c$, then $c = \frac{\log b}{\log a}$

The log graph: $y = 10^x$



You could also create a chart showing a few examples of conversions from logs to indices and from indices to logs, to help students to understand the principle.

Collect the financial pages from newspapers to give students real examples of stock market summaries in the capital markets section of the work.

Introduction for students

Write $8 = 2^3$ on the board and explain what a logarithm is by showing students that the log of a number is the index to which it is raised to get the answer. Here, $\log_2 8 = 3$.

Ask students to draw up a table of values in order to draw the graph $y = \log x$.

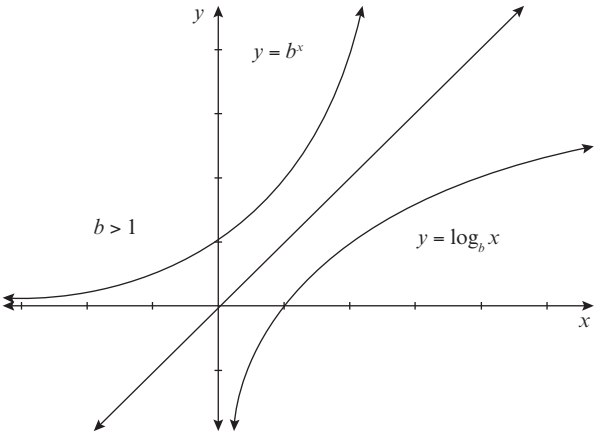
Give them the x values in the table and get them to work out the y values, which are given here:

x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1 000
y	–2	–1	0	1	2	3

Have graph paper available. Draw the log graph on graph paper.

Extension

You could point out that the log graph and the exponent graph are reflections of each other in the line $y = x$.



Introduce each section by going through the worked examples before asking students to complete the given Exercises.

Answers

Exercise 5.1

1. a) $\log_2 16 = 4$ **b)** $\log_3 27 = 3$ **c)** $\log_5 125 = 3$
d) $\log_6 36 = 2$ **e)** $\log_7 343 = 3$ **f)** $\log_4 8 = x$
g) $\log_s u = t$ **h)** $\log_5 1 = 0$ **i)** $\log_4 32 = x$
j) $\log_{64} 16 = x$

2. a) $4^2 = 16$ **b)** $6^3 = 216$ **c)** $4^{\frac{3}{2}} = 8$
d) $8^{\frac{2}{3}} = 4$ **e)** $16^{\frac{1}{2}} = 4$ **f)** $9^{\frac{3}{2}} = 27$
g) $s^u = t$ **h)** $10^0 = 1$ **i)** $5^x = 50$
j) $a^d = b^c$

3. a) $\log_7 49 = 2$ **b)** $\log_5 25 = 2$ **c)** $\log_2 64 = 6$
d) $\log_3 81 = 4$ **e)** $\log_4 64 = 3$ **f)** $\log_2 32 = 5$
g) $\log_3 243 = 5$ **h)** $\log_8 512 = 3$ **i)** $\log_9 729 = 3$
j) $\log_5 625 = 4$ **k)** $\log 10 = 1$ **l)** $\log 100 = 2$
m) $\log 1 = 0$ **n)** $\log 1\,000 = 3$ **o)** $\log \frac{1}{100} = -2$

Exercise 5.2

1. a) 0 **b)** 1.4 (accept 1.5) **c)** 1.6
d) 1.9

2. a) 1 **b)** 31.6 **c)** 63.1
d) 158.5

3. a) 1.3 **b)** 1.6 **c)** 1.8
d) 1.9

Exercise 5.3

1. a) 0.6020 **b)** 1 **c)** 0.9542
d) 1.1761 **e)** 1.0791 **f)** 0.1761
g) −0.097 **h)** 1.4313 **i)** 1.6532
j) −0.2219

2. a) $2 \log a$ **b)** $\log a + \log b$
c) $\log a + \log b + \log c$ **d)** $\log a - \log b$
e) $2 \log a + \log b$ **f)** $\frac{1}{2}\log a$
g) $(\log a + \log b) - \log c$ **h)** $\log a - (\log b + \log c)$
i) $2 \log a + 2 \log b + \log c$ **j)** $\frac{1}{2}(\log a + \log b) - \log c$

3. a) $\log a^3bc$ **b)** $\log \frac{a}{bc}$ **c)** $\log a^2b^2$

d) $\log (\sqrt{a})(b)$ **e)** $\log \frac{a^2}{b^3}$

4. a) 3 **b)** 5 **c)** 3

d) 1 **e)** –2

Exercise 5.4

1. a) $1\ 000 = 1 \times 10^3$ **b)** $32\ 560 = 3.256 \times 10^4$

c) $456.9 = 4.569 \times 10^2$ **d)** $69.98 = 6.998 \times 10^1$

2. a) 2 **b)** 1 **c)** 3

d) 0

3. a) 7 679 **b)** 9 694 **c)** 8 978

d) 0051

4. a) 2.8451 **b)** 1.8451 **c)** 3.5623

d) 1.5623

5. a) 10 000 **b)** 7 244 **c)** 17.18

d) 5.508

Exercise 5.5

1. a) 48.06 **b)** 184.6 **c)** 440.44

d) 1.632 **e)** 1 366.4 **f)** 20.098

g) 3 745.4 **h)** 1 088.8

2. a) 87.981 **b)** 5.753 **c)** 33.917

d) 1.281

3. a) 53.29 **b)** 22.7 **c)** 2.134

d) 1.061

Exercise 5.6

1. 9.5%

2. $A = 150\ 000(1 + 0.13)^3$

$A = ₦216\ 434.55$

3. $A = 530(1 + 0.0396) = \$551$ billion

4. ₦225 326 870

5. $358\ 600 = 220\ 000(1 + 0.13)^n$

$1.63 = 1.13^n$

$\therefore n = 4$ years

6. A = 87(1 + 0.087) = ₦94.57

7. 5 700 × 12.36 = 70 452
5 700 × 13.32 = 75 924
He made ₦5 472 profit when he sold the shares.

8. **a)** ₦2.64 **b)** a profit **c)** ₦23 250

Assess your progress

1. 0.9030

2. **a)** between 0 and 1 **b)** between 1 and 2
c) between 2 and 3 **d)** between 4 and 5
e) between 0 and −1 **f)** between −3 and −4
g) between 0 and −1 **h)** between −1 and −2
i) between 5 and 6 **j)** between −5 and −6

3. $k = 15$

4. **a)** $x = 3\frac{1}{2}$ **b)** $x = -2$ **c)** $x = \frac{3}{5}$
d) 4

5. **a)** $a + c$ **b)** $1 + 2a + b + c$ **c)** $\log \sqrt{2} + b + c$
d) $a + b - 3c$ **e)** $b + c - 2$

6. **a)** –2 **b)** 1 **c)** –2
d) $\frac{1}{4}$

7. $x = 2\frac{1}{3}$

8. $x = \frac{1}{3}$

9. **a)** $xy = 2^3$ **b)** $x = 4; y = 2$

10. $x = 3; y = 7$

11. $x = 3; y = 1$

12. **a)** 76.33 **b)** 1.7898 **c)** 13.27

TOPIC 6

Linear equations and variations

Introduction

This topic deals with changing the subject of a formula and solving linear equations. The necessary steps are given in point form and there are worked examples of the different types of problems.

Then we look at different types of variation: direct, inverse, joint and partial, with examples of each type. We see how variations are applied to everyday problems.

Common difficulties

Students sometimes mix up the steps for changing the subject of a formula. Remind them to get rid of fractions by finding the lowest common denominator of each side of the equation. When multiplying out brackets, be extra careful when there is a minus sign in front of the bracket.

Revise the rules of surds and indices.

Students must remember that $\sqrt{a^2 \times b^2} = a \times b$ but $\sqrt{a^2 + b^2} \neq a + b$.

Show them the example: $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$, so $\sqrt{3^2 + 4^2} \neq 3 + 4$.

Students need to read the questions carefully to decide which sort of variation is being described.

When changing the subject of a formula and the last step is to take the square root, students must remember that there are two potential answers (positive and negative). This does not apply when working with length, people, etc., as these can only be positive.

Preparation

Have a list of the formulae for surface area and volume. Use these to illustrate changing the subject of the formula.

You could also make a chart from the steps for changing the subject of an equation, and for solving a linear equation, as provided on the next page:

Important steps in changing the subject of an equation

- Get rid of root signs.
- Get rid of fractions by multiplying both sides of the equation by the LCD.
- Multiply out brackets.
- Gather all terms containing the required variable on one side of the equation.
- Simplify like terms.
- Divide both sides of the equation by the coefficient of the required variable.
- If the answer is a power, take roots both sides of the equation to get rid of the power.

Important steps in solving a linear equation

- Get rid of fractions by multiplying both sides of the equation by the LCD.
- Multiply out brackets.
- Gather all terms containing the required variable on one side of the equation.
- Simplify like terms.
- Divide both sides of the equation by the coefficient of the required variable.
- Simplify fractions where possible.

Prepare a chart of graphs to show how direct and indirect variation can be illustrated with straight lines or hyperbolas.

Introduction for students

Revise basic equation solving with students. Discuss the number of potential answers to a linear equation. Give an example of a linear equation with no solution, one with only one solution and an example of an identity that is true for any real value of x.

Go through the important steps to take to solve a linear equation.

Changing the subject of a formula is a very important tool in mathematics. Go through the steps to take to change the subject of a formula and the worked examples.

Discuss direct and indirect variation and explain partial and joint variation, giving examples of each type of problem.

Answers

Exercise 6.1

1. $b = 5a + 7$

2. $z = \frac{6y - x}{6}$ or $z = y - \frac{x}{6}$

3. $c = \pm \frac{a}{4b}$

4. $n = \frac{360}{180 - a}$

5. $a = \frac{b^2 - d^2}{4c}$

6. $b = \pm \sqrt{a - 2c}$

7. $r = \sqrt{\frac{\mathrm{V}}{\pi h}}$

8. $h = \frac{\mathrm{A} - 2\pi r^2}{2\pi r}$

9. $b = \frac{3 + 3a}{a - 1}$

10. $w = \pm \sqrt{x^2 - y^2}$

11. $p = \frac{mn}{m - n}$

12. $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$

13. $\mathrm{H} = \frac{4k\pi^2}{\mathrm{MT}^2}$

14. $y = \frac{1}{x - 1}$

15. $d = \frac{2s - 2an}{n(n - 1)}$

Exercise 6.2

1. $x = -3$

2. $x = 3$

3. $x = -10$

4. $x = 12$

5. $x = -1$

6. $x = -5$

7. $x = \frac{6}{5}$

8. $x = 11$

9. $x = -\frac{5}{4}$

10. no solution

Exercise 6.3

1. $\mathrm{A} = b^2 \therefore \mathrm{A} \propto b$: The area of a square varies directly as the length of the side b varies.

2. a) $9 = k4$
$\therefore k = \frac{9}{4}$
$\therefore y = \frac{9}{4}x$

b) $y = \frac{9}{4}(12)$
$\therefore y = 27$

Exercise 6.4

1. a) $y \propto x$
$y = kx$
$7 = k4$
$\therefore k = \frac{7}{4}$
$\therefore y = \frac{7}{4}x$

b) $\frac{1}{2} = \frac{7}{4}x$
$\therefore x = \frac{2}{7}$

2. a) $p \propto q^2$

$p = \frac{2}{5}q^2$

$\therefore p = \left(\frac{2}{5}\right)(5^2)$

$\therefore p = 10$

b) $160 = \frac{2}{5}q^2$

$\therefore q = \pm 20$

3. a) $A \propto r^2$; $A = 4\pi r^2$

b) $A = (4\pi)\left(\frac{1}{2}\right)^2$

$\therefore A = \pi$

c) $16\pi r^2 = A$

$\therefore r^2 = \frac{A}{16\pi}$

$\therefore r = \frac{1}{4}\sqrt{\frac{A}{\pi}}$

4. $m = \frac{1}{6}n$

$m = \left(\frac{1}{6}\right)(42)$

$\therefore m = 7$

5. a) $r = 5t$

b) $r = 5(3)$

$\therefore r = 15$

c) $200 = 5t$

$\therefore t = 40$

6. $h = 0.42t$

$6.3 = 0.42t$

$\therefore t = 15$

It takes 15 seconds to fill the can to a height of 6.3 cm.

7. $V = \frac{1}{8}r^3$

$\mathbf{V} = \left(\frac{1}{8}\right)(10^3)$

$\mathbf{V} = 125 \text{ cm}^3$

8. a) $y = 10x^2$

b) $y = (10)(5^2)$

$\therefore y = 250$

Exercise 6.5

1. a) $m = \frac{10}{n}$

b) $m = \frac{10}{10}$

$\therefore m = 1$

c) $6 = \frac{10}{n}$

$\therefore n = \frac{5}{3}$

2. $y = \frac{40}{x}$

$\therefore a = 8$

$\therefore b = 2$

Exercise 6.6

1. $y \propto \frac{1}{(x-4)^2}$

$y = \frac{k}{(x-4)^2}$

$\therefore (x-4)^2 = \frac{k}{y}$

2. a) $(2-4)^2 = \frac{k}{5}$

$\therefore k = 20$

$\therefore (x-4)^2 = \frac{20}{y}$

b) $(5-4)^2 = \frac{20}{y}$

$\therefore y = 20$

c) $(x-4)^2 = \left(\frac{20}{\frac{1}{5}}\right)$

$\therefore x = 14$ or $x = -6$

3. a) $v = \frac{k}{u^2}$

$u^2 = \frac{k}{v}$

$5^2 = \frac{k}{10}$

$\therefore k = 250$

$\therefore v = \frac{250}{u^2}$

b) $v = \frac{250}{10^2}$

$\therefore v = \frac{5}{2}$

c) $u^2 = \frac{250}{10}$

$\therefore u = \pm 5$

Exercise 6.7

1. $r \propto \frac{1}{\sqrt{t}}$

$\therefore r = \frac{k}{\sqrt{t}}$

2. a) $4 = \frac{k}{\sqrt{q}}$

$k = 12$

$\therefore r = \frac{12}{\sqrt{t}}$

b) $r = \frac{12}{\sqrt{25}}$

$\therefore r = \frac{12}{5}$

c) $8 = \frac{12}{\sqrt{t}}$

$\sqrt{t} = \frac{3}{2}$

$\therefore t = \frac{9}{4}$

3. a) $x = \frac{2}{\sqrt{y}}$

b) $x = \frac{2}{\sqrt{9}}$

$\therefore x = \frac{2}{3}$

c) $12 = \frac{2}{\sqrt{y}}$

$\therefore y = \frac{1}{36}$

Exercise 6.8

1. $a = \frac{693}{b}$

$x = 9$

$y = 63$

2. a) $y = \frac{40}{x}$

b) $y = \frac{40}{10}$

$\therefore y = 4$

c) $2 = \frac{40}{x}$

$\therefore x = 20$

3. a) $y = \frac{10}{x^2}$

$\therefore x^2 = \frac{10}{y}$

b) $\left(\frac{1}{3}\right)^2 = \frac{10}{y}$

$\therefore y = 90$

c) $x^2 = \frac{10}{1.6}$

$\therefore x = \pm\frac{5}{2}$

4. a) $x = \frac{6}{\sqrt{y}}$

$x = \frac{6}{\sqrt{16}}$

$\therefore x = \frac{3}{2}$

b) $\frac{1}{2} = \frac{6}{\sqrt{y}}$

$\therefore y = 144$

5. a) $y = \frac{9}{x + 8}$

b) $y = \frac{9}{10 + 8}$

$\therefore y = \frac{1}{2}$

6. a) $(p - 12)^2 = \frac{13}{z}$

b) $(9 - 12)^2 = \frac{13}{z}$

$\therefore z = \frac{13}{9}$

c) $(p - 12)^2 = \left(\frac{13}{\frac{1}{13}}\right)$

$\therefore p = 25$ or $p = -1$

7. $p = \frac{10}{\sqrt{q}}$

$100 = \frac{10}{\sqrt{q}}$

$\therefore q = \frac{1}{100}$

8. $y = \frac{16}{x^3}$ or $x = 2 \,.\, \sqrt[3]{2}\ \sqrt[3]{y}$

$y = \frac{16}{64}$

$\therefore y = \frac{1}{4}$

9. $y = \frac{128}{(a + 3)^3}$ or $a + 3 = 4 \,.\, \sqrt[3]{2}\ \sqrt[3]{y}$

$y = \frac{128}{(4 + 3)^3}$

$\therefore y = \frac{128}{343}$

Exercise 6.9

1. a) $P = kQR$ **b)** $k = \frac{1}{6} \therefore Q = 4$

2. $A = \frac{kB^2}{C} \therefore k = 16 \therefore B = \pm 5$

3. a) $M = \frac{kN}{P}$ **b)** $k = \frac{1}{2} \therefore M = 2$

4. $A = kB\sqrt{C} \therefore k = -3 \therefore B = -\frac{5}{3}$

5. $V = \frac{kh}{c^2} \therefore k = \frac{81}{5} \therefore h = 247$ m

6. $M = \frac{12N}{P^2}$

7. a) $n = a + km^2$

b)
$11 = a + k$ ①
$5 = a + 4k$ ②
① − ②: $6 = -3k$
$k = -2$
$\therefore a = 13$
$\therefore n = -19$

8.
$r = a$
$350 = a + k2\,500$ ①
$190 = a + k900$ ②
① − ②: $160 = k1\,600$
$k = \frac{1}{10}$
$\therefore a = 100$
$\therefore v = 45\ km/h$

9. a) $C = A + kE$

b)
$7\,420 = A + k500$ ①
$7\,604 = A + k600$ ②
② − ①: $184 = k100$
$k = 1.84$
$\therefore A = 6\,500$
$\therefore C = ₦7\,512$

10.
$C = a + ke$
$28\,295 = a + k60$ ①
$32\,495 = a + k75$ ②
② − ①: $4\,200 = 15k$
$k = 280$
$\therefore a = 11\,495$
$\therefore C = 11\,495 + 280(80)$
$= ₦33\,895$

Assess your progress

1. a) $a = \frac{b(x+y)}{(x-y)}$ **b)** $a = T_n - dn + d$ **c)** $a = \frac{xy}{x+y}$

d) $a = n - \sqrt{m}$ **e)** $a = \frac{xy}{x-y}$ **f)** $a = \frac{3cx^2}{b}$

2. a) $x = 21$ **b)** $x = -\frac{13}{2}$ **c)** $x = \frac{18+7y}{4}$

d) $x = 4$

3. $y = \frac{16}{9}$

4. $y = 108$

5. $y = 80$

6. $b = 189$

7. $x = 2$

8. a) $C = a + kT$

b)
$$4\,200 = a + 3k \ ①$$
$$6\,600 = a + 5k \ ②$$
$$② - ①: \ 2\,400 = 2k$$
$$k = 1\,200$$
$$\therefore a = 600$$
$$\therefore T = 7\frac{1}{2} \text{ hours}$$

9. $D = \frac{k}{h}$

$k = 5\,376 \therefore 32$ days

10. $k = 5\,000 \therefore$ ₦5 000 000

11. $k = 15\,000 \therefore T = 25$ tickets

TOPIC 7

Quadratic equations

Introduction

In this topic we will first revise methods of factorising expressions. We look at the three main types of factorising: taking out the highest common factor, the difference of two squares, and trinomials as well as grouping. Then we look at solving quadratic equations using factorising.

We introduce the method of completing the square to solve quadratic equations that do not factorise easily. Then we use the method of completing the square to derive the quadratic formula.

Then we see how to draw quadratic functions and how to read values from a graph. We note the effect of changes of a, b and c in the quadratic function $y = ax^2 + bx + c$.

Note that in SS1 students do not need to know the term 'parabola' so we have called it a quadratic function.

Common difficulties

Students battle to factorise trinomials as they do not know how to write numbers as a product of their factors. It is very important that they know their times tables well and understand how to write a number as a product of its factors. For example $12 = 12 \times 1$ or 6×2 or 3×4, so they need to look at all possible options available. They often do not recognise when there is a switch-around: $b - a = -(a - b)$.

Students need to understand what a quadratic graph looks like. Remember that when $a < 0$, the arms of the graph face downwards and when $a > 0$, the arms of the graph face upwards.

Preparation

Have a list of the different methods of factorising with examples.

Have graph paper available for students to use.

Prepare a chart of a variety of linear and quadratic graphs to illustrate the difference between a straight line graph obtained from a linear function and the graph obtained from a quadratic function.

Introduction for students

Revise factorisation and make sure that students can write numbers as a product of their factors in different ways, for example: $24 = 24 \times 1$ or 12×2 or 8×3 or 6×4.

Discuss the number of potential answers to a quadratic equation. Give an example of a quadratic equation with no solution, one with only one solution and one with two different solutions. Explain that the solutions to the equation are also called the roots of the equation.

Go through the different ways to solve a quadratic equation. Explain that you will be using quadratic equations to draw different quadratic graphs, and discuss why the roots of the equation give the x intercepts of the quadratic function.

Answers

Exercise 7.1

1. **a)** $5(2x - 7)$ **b)** $c(c - 1)$ **c)** $ab(a + b)$
d) $5(2x^2 - 3y^2)$ **e)** $3(y^2 + 4y - 2)$ **f)** $2ab(1 - 4a - 2b)$
g) $5(p - 2q + 3r)$ **h)** $2(x^2 + 3x - 1)$ **i)** $9(2x^2 - 1)$
j) $4(2r^2 - 4r + 1)$

2. **a)** $(x - 5)(x + 5)$ **b)** $(c - 1)(c + 1)$ **c)** $(2a - 3)(2a + 3)$
d) $(x - y)(x + y)$ **e)** $(4y - 5)(4y + 5)$ **f)** $(12 - a)(12 + a)$
g) $(7 - d)(7 + d)$ **h)** $(2x - 1)(2x + 1)$ **i)** $2(x - 3)(x + 3)$
j) $2(2r - 1)(2r + 1)$

3. **a)** $(x + 5)^2$ **b)** $(a + 2)^2$ **c)** $(a - 3)^2$
d) $(x - 5)(x + 4)$ **e)** $(y - 5)(y - 2)$ **f)** $(x + 4)(x - 3)$
g) $(d + 5)(d - 3)$ **h)** $(x + 6)(x - 2)$ **i)** $2(x - 12)(x - 1)$
j) $(r - 6)^2$

4. **a)** $2x(x + 10)$ **b)** $(3x + 1)(1 - 2y)$
c) $(x - 4)^2$ **d)** $3(x - 5)(x + 4)$
e) $(3y - 5z)(x - 3y)$ **f)** $2(y - 4)(y - 3)$
g) $3(x + 5)(x - 3)$ **h)** $4(a + 2)(a + 3)$
i) $(2x + 3y - 5)(2x + 3y + 5)$ **j)** $(x^2 + 1)(x - 1)(x + 1)$

Exercise 7.2

1. a) $x = \pm 1$ **b)** $x = 1$ **c)** $x = 0$ or $x = 12$

d) $x = 0$ or $x = 2$ **e)** $x = -3$ or $x = -4$ **f)** $x = \pm 7$

g) $x = \pm \frac{1}{2}$ **h)** $x = 2$ **i)** $x = \pm 3$

j) $x = 0$ or $x = 2$

2. a) $x = \pm 5$ **b)** $x = \pm 2$ **c)** $x = \pm \frac{3}{2}$

d) $x = \pm 2$ **e)** $x = \pm \frac{5}{4}$ **f)** $x = \pm 11$

g) $x = \pm 8$ **h)** $x = \pm \frac{1}{2}$ **i)** $x = \pm 4$

j) $x = \pm \frac{6}{5}$

3. a) $x = -5$ **b)** $x = 1$ or $x = -\frac{1}{2}$ **c)** $x = 1$ or $x = 5$

d) $x = -\frac{1}{2}$ or $x = 5$ **e)** $x = -1$ or $x = 7$ **f)** $x = -5$ or $x = 6$

g) $x = -5$ or $x = 3$ **h)** $x = 2$ or $x = -6$ **i)** $x = 1$ or $x = 12$

j) $x = \frac{3}{2}$ or $x = -2$

4. a) $x = 12$ or $x = -2$ **b)** $x = \frac{5}{3}$ or $x = -1$

c) $x = -\frac{2}{5}$ or $x = -\frac{1}{2}$ **d)** $x = -4$ or $x = 5$

e) $x = \frac{2}{3}$ **f)** $x = 3$ or $x = 4$

g) $x = 2$ or $x = -\frac{1}{2}$ **h)** $x = \frac{3}{2}$ or $x = -\frac{1}{4}$

Exercise 7.3

1. a) $(x + 3)^2$ **b)** $(x - 4)^2$ **c)** $\left(x + \frac{1}{2}\right)^2 - \frac{17}{4}$

d) $-(x + 1)^2 + 4$ **e)** $2\left(x + \frac{1}{4}\right)^2 - \frac{33}{8}$ **f)** $-3(x - 1)^2 + 4$

2. a) $\left(x - \frac{b}{2}\right)^2 + 1 - \frac{b^2}{4}$ **b)** $a\left(x + \frac{2}{a}\right)^2 - 2 - \frac{4}{a}$

c) $a\left(x + \frac{1}{a}\right)^2 - 3 - \frac{1}{a}$ **d)** $-2a\left(x + \frac{1}{a}\right)^2 + 6 + \frac{2}{a}$

3. a) minimum **b)** maximum **c)** minimum

d) maximum

4. a) $(-2; 0)$ **b)** $(-5; 0)$ **c)** $\left(-\frac{1}{2}; -8\frac{1}{2}\right)$

d) $(-1; 12)$

Exercise 7.4

1. a) $x = -2$ **b)** $x = 5$ **c)** $x = -1$ or $x = 9$
d) $x = 1$ or $x = 5$

2. a) $x = \frac{5 \pm \sqrt{13}}{2}$ **b)** $x = \frac{3 \pm \sqrt{5}}{2}$ **c)** $x = 3 \pm \sqrt{2}$
d) $x = 1 \pm \frac{\sqrt{x^2 - 4a}}{2a}$

3. a) $x = 2.62$ or $x = 0.38$ **b)** $x = -0.59$ or $x = -3.41$
c) $x = 0.81$ or $x = 6.19$ **d)** $x = -6.24$ or $x = 0.24$
e) $x = 0.34$ or $x = -1.09$ **f)** $x = 2.23$ or $x = -0.90$

Exercise 7.5

1. a) $x = -2 \pm \sqrt{10}$ **b)** $x = 3 \pm 2\sqrt{5}$ **c)** $x = -4 \pm \sqrt{7}$
d) $x = 2(1 \pm \sqrt{2})$ **e)** $x = \frac{-b \pm \sqrt{b^2 + 4c}}{2}$ **f)** $x = \frac{b \pm \sqrt{b^2 + 4ac}}{-2a}$

2. a) $x = 4.30$ or $x = 0.70$ **b)** $x = 4.19$ or $x = -1.19$
c) $x = 0.59$ or $x = 3.41$ **d)** $x = 0.77$ or $x = -0.43$
e) $x = 2.62$ or $x = 0.38$ **f)** $x = -0.45$ or $x = 4.45$

3. a) $x = -2$ or $x = \frac{3}{2}$ **b)** $x = -0.59$ or $x = -3.41$
c) $x = -1.45$ or $x = 3.45$ **d)** $x = 4.30$ or $x = 0.70$
e) $x = -6.24$ or $x = 0.24$ **f)** $x = 0.34$ or $x = -1.09$

4. a) $x = -3$ or $x = 2$ **b)** $x = 1.56$ or $x = -2.56$
c) $x = 1.54$ or $x = -4.54$ **d)** $x = 0.55$ or $x = 5.45$
e) $x = \frac{1}{3}$ or $x = -5$ **f)** $x = 1.36$ or $x = -7.36$

Exercise 7.6

1. $y = \frac{1}{2}x^2 + x - 12$

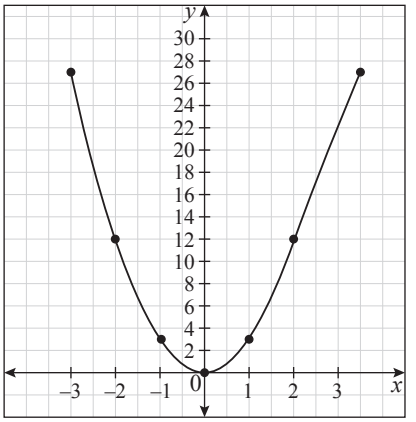
2. $y = x^2 - 4x - 5$

3. $y = 2x^2 + 4x - 6$

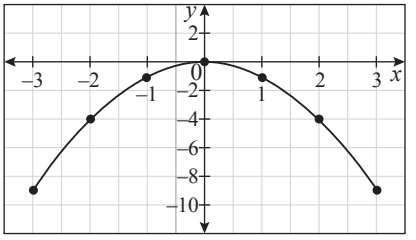
4. $y = -x^2 - 3x$

Exercise 7.7

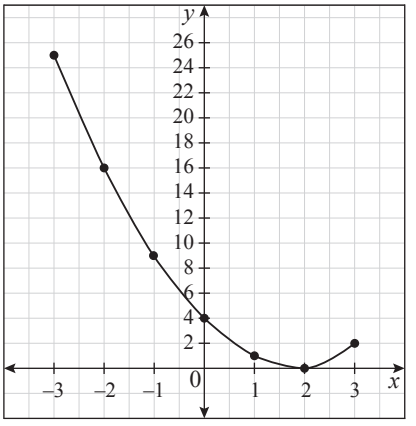
1. a)



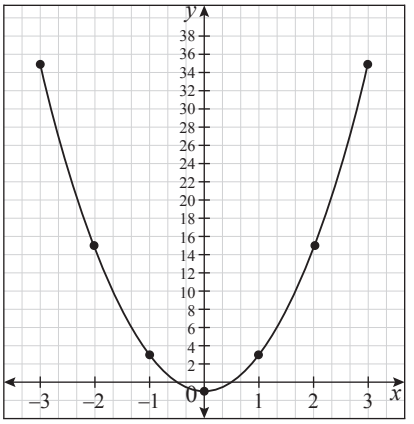
b)



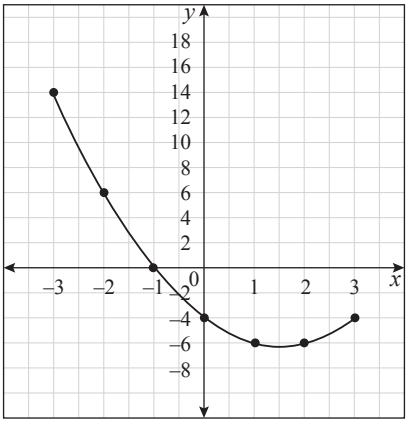
c)



d)



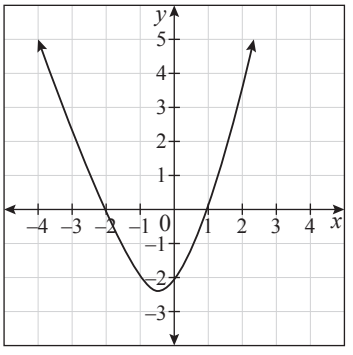
e)



2. a) $y = -2$ **b)** $x = 1$ or $x = -2$

c) $x = -\frac{1}{2}$ **d)** up

e)

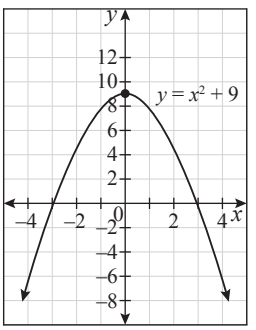


3. a) $x = \frac{3}{2}$ **b)** $x = 3$ **c)** $x = 0$
d) $x = \frac{7}{6}$

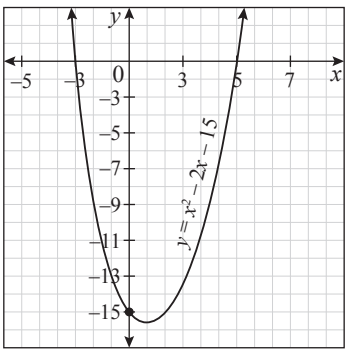
4. a) $\left(\frac{3}{2}; -6\frac{1}{4}\right)$ **b)** $(3; 4)$ **c)** $(0; -8)$
d) $\left(\frac{7}{6}; -2\frac{1}{12}\right)$

5. a) up **b)** down **c)** up
d) down

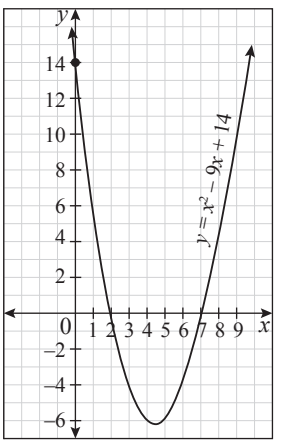
6. a)



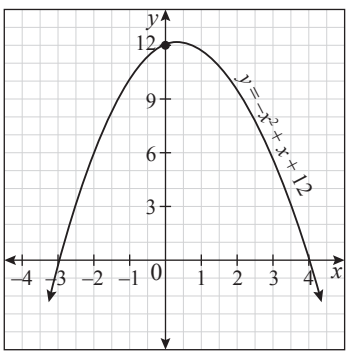
c)



b)



d)



Exercise 7.8

1. 8 m × 6.5 m
2. $x = 13$
3. **a)** 3 m × 6 m
b) 18 m^2
4. 4 and 9
5. 60 km/h
6. 40 days and 60 days

Assess your progress

1. **a)** $(3x + 2)(2x - 5)$ **b)** $(4x - 3)(4x + 3)$ **c)** $3(x + 4)(x - 2)$

2. **a)** $(x - 5)^2 - 5$ **b)** $(x - 1)^2 - 5$ **c)** $2\left(x - \frac{3}{4}\right)^2 - 9\frac{1}{8}$

3. **a)** $x = \frac{3 \pm \sqrt{41}}{2}$ **b)** $x = \frac{2 \pm \sqrt{10}}{3}$ **c)** $x = 3 \pm 2\sqrt{3}$

4. **a)** $x = 0$ or $x = \frac{5}{3}$ **b)** $x = 1.74$ or $x = -6.34$
c) $x = 13$ or $x = -15$ **d)** $x = 1$ or $x = -\frac{2}{3}$

5. ₦87

6. 125 m × 25 m

7. 23 and 17

8. 9 days and 18 days

9. 3 km/h

10. 150 m × 80 m

TOPIC 8

Sets

Introduction

We first look at the meaning of a set in mathematics and the mathematical language used to describe sets.

Then we study different types of sets: null and unit sets, disjoint sets, equal and equivalent sets, with examples of each type. We also study subsets and power sets.

We use Venn diagrams to show the relationships between sets and subsets in a universal set. We also use Venn diagrams to illustrate intersection or union of sets. We use the $\cap$ symbol for intersection and the $\cup$ symbol to indicate the union of sets.

Finally, we use Venn diagrams to solve problems.

Common difficulties

Students get the different types of sets confused. There are many new words in this section of work. Encourage the students to look again at the explanations provided whenever they are not sure of the meaning of a word.

Explain the difference between union of sets (using the $\cup$ symbol) and the intersection of sets (using the $\cap$ symbol).

Emphasise the importance of using the correct notation. For example, $n(A)$ = the number of elements in set A. If set A = $\{3, 6, 9\}$, then students need to be clear that 3, 6 and 9 are three elements in set A but $\{3\}$, $\{6\}$ and $\{9\}$ are three subsets of set A.

Show how the different types of sets can be illustrated using Venn diagrams.

Preparation

Prepare a list of the different types of sets and illustrate them on a chart to have on the classroom wall. Bring in pictures of different sets of objects found in everyday life.

You can make a chart of the different symbols that are used in set theory for the classroom wall for reference.

{…}	the set of … (when all elements of a set are listed, they are enclosed by curly brackets)
ξ	the universal set
$\in$	belongs to (is an element of)
$\notin$	does not belong to (is not an element of)
$\subset$	is a subset of (is included within)
$\not\subset$	is not a subset of
$\subseteq$	improper subset
$\cup$	union
$\cap$	intersection
A' or A^C	the complement of A (the set of all elements in the universal set that are not in A)
$=$	is equal to
$\equiv$	is equivalent to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$n(A)$	the cardinal number of A (the number of elements in a set A)
$\varnothing$ or { }	the null or empty set
$\{x: x \text{ is } \ldots\}$	the set of all elements x of a set such that x is … (used to describe a particular property of x that means it belongs to this set, e.g. 'x is an integer from –10 to –3' is written as $\{x: -10 \leq x \leq -3, x \in \mathbb{Z}\}$)

Introduction for students

Discuss the photographs in the Student's Book of goods for sale that are grouped into sets. Ask the students why traders arrange their goods in sets. What are the advantages of good grouping and what are the disadvantages of having goods displayed in a disorganised way?

Go through the notes about the different types of sets. Discuss all the symbols used in this section and notice how many you have used before in algebra.

Make sure students understand the use of the terms rules, lists and roster, and that we will focus on set builder notation.

Answers

Exercise 8.1

1. a) Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday
b) 1, 2, 3, 4, 6, 8, 12, 24
c) 4, 8, 12, 16, 20, 24

2. a) the first six odd numbers
b) the first five prime numbers
c) planets in our Solar System

3. a) $n(P) = 3$
b) {4, 6, 9}, {4, 6}, {6, 9}, {4, 9}, {4}, {6}, {9}, { }
c) finite

4. a) $18 \notin \{\text{prime numbers}\}$
b) $91 \in \{\text{multiples of 7}\}$
c) Igbo $\notin$ {languages spoken in Korea}
d) $144 \in \{\text{square numbers}\}$

Exercise 8.2

1. Student's own work

2. A: $n(A) = 4$
B: Student's own work
C: Student's own work
D: $n(D) = 500$
E: $n(E) =$ infinite
F: $n(F) = 6; x > 0$
G: $n(G) = 6$

3. A: Finite **B:** Finite **C:** Infinite
D: Finite **E:** Finite **F:** Infinite

4. A: Singleton
B: Empty
C–F: C, D E and F are equivalent sets; D and F are equal sets

5. B and C

Exercise 8.3

1. a) $\subset$ **b)** $\not\subset$ **c)** $\supset$
d) $\not\subset$ **e)** $\subset$ **f)** $\supset$
g) $\supset$

2. a) $n(\mathrm{P}) = 2^0 = 1$ **b)** $n(\mathrm{Q}) = 2^1 = 2$ **c)** $n(\mathrm{R}) = 2^2 = 4$
d) $n(\mathrm{S}) = 2^3 = 8$ **e)** $n(\mathrm{T}) = 2^4 = 16$

3. a) (i) $\{a, b\}, \{a\}, \{b\}, \{\ \}$
(ii) $\{a, b, c\}, \{a, c\}, \{a, b\}, \{b, c\}, \{a\}, \{b\}, \{c\}, \{\ \}$
(iii) $\{a, b, c, d\}, \{a\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, c\}, \{a, b\}, \{b, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{\ \}$
(iv) $\{a, b, c, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d , e\}, \{a\}, \{b\}, \{c\}, \{d\}, \{e\},\{\ \}$

b)

Number of elements in a set	0	1	2	3	4	5
Number of subsets	1	2	4	8	16	32

c) $2^{n(\mathrm{P})}$

Exercise 8.4

1. a) $\mathrm{A} \cup \mathrm{B} = \{2, 4, 5, 8, 16\}$
b) $\mathrm{B} \cup \mathrm{C} = \{2, 4, 7, 8, 9, 15, 16\}$
c) $\mathrm{A} \cup \mathrm{C} = \{4, 5, 7, 8, 9, 15, 16\}$
d) $\mathrm{A} \cup \mathrm{B} \cup \mathrm{C} = \{2, 4, 5, 7, 8, 9, 15, 16\}$

2. a) $\mathrm{X} \cup \mathrm{Y} = \{1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$
b) $\mathrm{X} \cup \mathrm{Z} = \{1.5, 2, 2.5, 3, 3.5, 4.5, 5, 5.5, 6, 6.5\}$
c) $\mathrm{Y} \cup \mathrm{Z} = \{3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5\}$
d) $\mathrm{X} \cup \mathrm{Y} \cup \mathrm{Z} = \{1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5\}$

3. a) $P \cup R = \{1, 3, 5, 7, 15, 21, 65, 147\}$
b) $R \cup Q = \{1, 2, 3, 4, 5, 8, 15, 32, 65\}$
c) $Q \cup P = \{2, 3, 4, 7, 8, 21, 32, 147\}$
d) $R \cup P \cup Q = \{1, 2, 3, 4, 5, 7, 8, 15, 21, 32, 65, 147\}$

4. $n(P \cup R) = 8$
$n(P) + n(R) = 4 + 5 = 9$
$n(P \cap R) = 1$
$n(P \cup Q) = 8$
$n(P) + n(Q) = 4 + 4 = 8$
$n(P \cap Q) = 0$
$n(Q \cup R) = 9$
$n(Q) + n(R) = 4 + 5 = 9$
$n(Q \cap R) = 0$
General rule: $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Exercise 8.5

1. a) $A \cap B = \{2, 4, 6\}$ **b)** $B \cap C = \{2, 4, 8\}$
c) $A \cap C = \{1, 2, 4\}$ **d)** $A \cap B \cap C = \{2, 4\}$

2. a) $X \cap Y = \{-5\}$ **b)** $X \cap Z = \{-3, -1\}$
c) $Y \cap Z = \{-9\}$ **d)** $X \cap Y \cap Z = \{\ \}$

3. a) $P \cap R = \{a, e\}$ **b)** $R \cap Q = \{a, g, o, n\}$
c) $Q \cap P = \{a\}$ **d)** $R \cap P \cap Q = \{a\}$

4. a) V and O are disjoint
b) V and P are not disjoint. They both contain the element 2.
c) O and P are not disjoint. They both have an infinite number of elements. For example, 3, 5, 7, 11, 13, 17, ... are elements of both sets.

Exercise 8.6

1. a) 46 **b)** 74 **c)** 17
d) 12 **e)** 94

2. 22 **3.** 38 **4.** 181

5. 4 **6.** 56 **7.** 27

8. 218

Exercise 8.7

1. {cats, dogs, chickens, goats, rabbits, sheep}

2. a) {Jan, Feb, Mar, Apr, May, June, July, Aug, Sep, Oct, Nov, Dec}
b) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31}

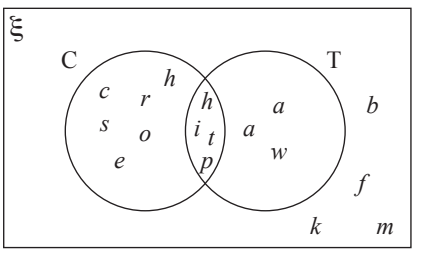
3. a) {Igbo, Yoruba, Fulfulde, Kanuri, Hausa, Ibibio]
b) $2^6 = 64$

4. a) $n(\xi) = 67$
b) $n(\xi) = 8$

5. J = {2, 3, 5, 7, 11, 13}
L = {3, 6, 9, 12, 15}
M = {1, 2, 4, 8, 16}
ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 15, 16}

Exercise 8.8

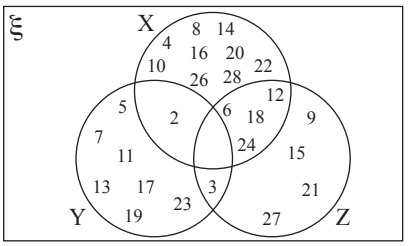
1. a)



b) C ∩ T= {*t*, *p*, *i*, *h*}
c) C ∪ T = {*c*, *h*, *r*, *i*, *s*, *t*, *o*, *p*, *h*, *e*, *r, a, w, a*}

2. ξ = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}
X = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28}
Y = {2, 3, 5, 7, 11, 13, 17, 19, 23}
Z = {3, 6, 9, 12, 15, 18, 21, 24, 27}

a)

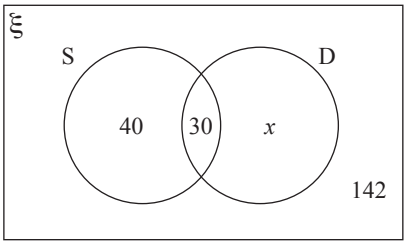


b) (i) $X \cap Y = \{2\}$

(ii) $Y \cap Z = \{3\}$

Exercise 8.9

1.

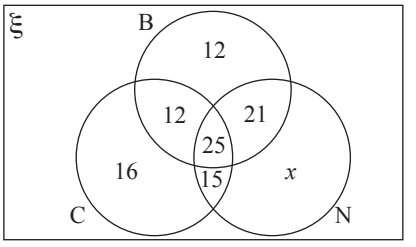


$40 + 30 + x + 142 = 272$

$\therefore x = 60$

$\therefore$ 90 students belong to the Debating Society.

2.



12 tourists visit Benin only

16 tourists visit Cameroon only

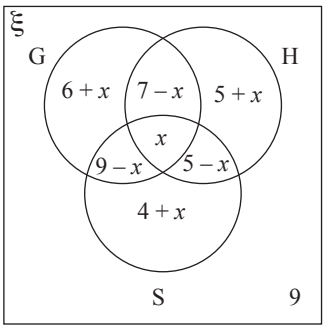
x tourists visit Nigeria only

$x + 15 + 25 + 21 + 12 + 12 + 16 = 120$

$\therefore x = 19$

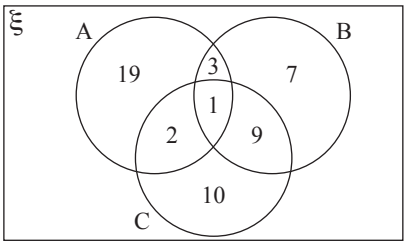
$\therefore$ 80 tourists visited Nigeria.

3.



$x = 3$

4.



The number of radios reported faulty adds up to 51 (not 45!).

Exercise 8.10

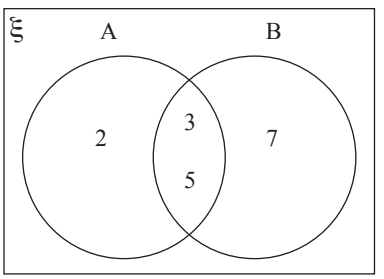
1. a) $\{c, e, f, g, h, i\}$ **b)** $\{a, b, f, g, h\}$
c) $\{b, d, f, h\}$ **d)** $\{a, b, c, d, e, f, g, h, i\}$
e) $\{\ \}$

2. a) all positive odd numbers
b) empty set
c) all composite numbers
d) positive integers greater than 9

3. A' = {5, 15, 25, 35...}, so A' = {all positive multiples of 5 that are not multiples of 10}

Exercise 8.11

1. a) A = {2, 3, 5}; B = {3, 5, 7}
b) A' = {7}; B' = {2}
c)



d) $A \cap B = \{3, 5\}$
$A \cup B = \{2, 3, 5, 7\}$

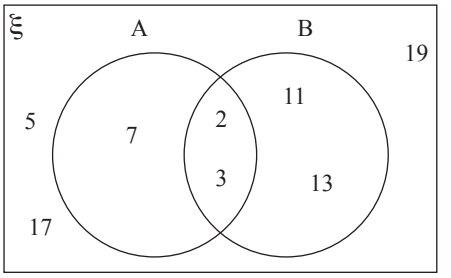
e) HFC = 15
LCM = 210

f) $C = \{3, 5\}$
$D = \{2, 3, 5, 7\}$

g) $C = A \cap B$; $D = A \cup B$

2. a) $\xi = \{2, 3, 5, 7, 11, 15, 17, 19\}$; $A = \{2, 3, 7\}$;
$B = \{2, 3, 11, 13\}$

b)



c) $A \cap B = \{2, 3\}$
$A \cup B = \{2, 3, 7, 11, 13\}$
$(A \cap B)' = \{5, 7, 11, 13, 17, 19\}$
$(A \cup B)' = \{5, 7, 19\}$
$A' \cap B' = \{5, 7, 19\}$
$A' \cup B' = \{5, 7, 11, 13, 17, 19\}$
$\therefore (A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

d) HCF = 6; LCM = 36 036

Assess your progress

1. a) {12, 23, 34}, {12, 23}, {23, 34}, {12, 34}, {12}, {23}, {34}, { }
b) {12, 23}, {23, 34}, {12, 34}, {12}, {23}, {34}

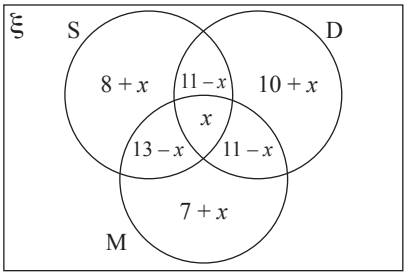
2. a) T **b)** F (1 is not a prime number)
c) T **d)** T
e) F **f)** T

3. a) $J \cup K = \{1, 2, 3, 4, 7, 8, 9, 10, 13, 14, 15, 16, 18\}$
b) $K \cup L = \{1, 2, 3, 7, 9, 13, 15\}$
c) $J \cup L = \{2, 3, 4, 7, 8, 10, 13, 14, 16, 18\}$
d) $J \cup K \cup L = \{1, 2, 3, 4, 7, 8, 9, 10, 13, 14, 15, 16, 18\}$
e) $(J \cup K)' = \{ \}$
f) $J' \cup K' = \{1, 2, 3, 4, 7, 8, 9, 10, 13, 14, 15, 16, 18\}$
g) $J \cap K = \{ \}$
h) $K \cap L = \{3, 7, 13\}$
i) $J \cap L = \{2\}$
j) $J \cap K \cap L = \{ \}$
k) $(K \cap L)' = \{1, 2, 4, 8, 9, 10, 14, 15, 16, 18\}$
l) $K' \cap L' = \{4, 8, 10, 14, 16, 18\}$

4. a) not disjoint, 14 and 28 are elements of both sets
b) disjoint
c) not disjoint, 7 is an element of both sets

5. 11

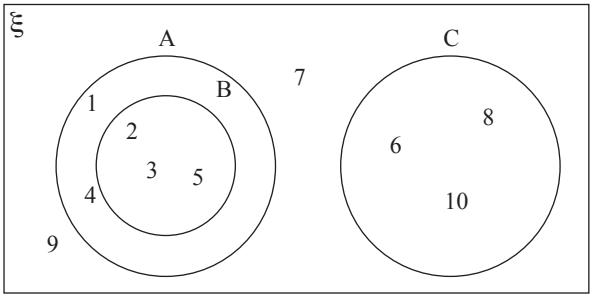
6. Let the number of students who took part in all three categories be x.



Students who only sang: $32 - (11 - x) - (13 - x) - x = 8 + x$
Students who only danced: $32 - (11 - x) - (11 - x) - x = 10 + x$
Students who only played musical instrument:
$31 - (11 - x) - (13 - x) - x = 7 + x$
$\therefore 8 + x + 10 + x + 7 + x + 11 - x + 13 - x + 11 - x + x = 65$
$60 + x = 65$
$\therefore x = 5$

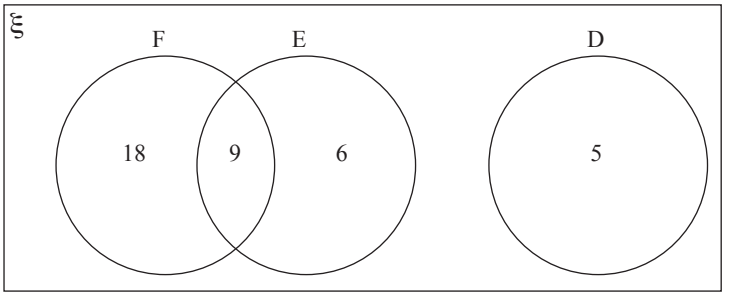
7. a) (i) A = {5, 10, 15, 20} **(ii)** B = {4, 8, 12, 16, 20}
(iii) C = {3, 6 ,9, 12, 15, 18}
b) (i) $A \cap B = \{20\}$ **(ii)** $A \cap C = \{15\}$
(iii) $B \cup C = \{3, 4, 6, 8, 9, 12, 15, 16, 18, 20\}$
c) Students' own work

8. a)



b) $n(A \cap B)' = 7$
c) $n(A \cap B') = 2$

9. a)



b) (i) 33
(ii) 18

10. a) $F \cap T \cap C = 4$
b) $T \cap C = 4$ and $2x$
c) $(F \cap T) - (F \cap T \cap C) = 5$
d) $n(C) = 17$
e) $n(T' \cup F) = 27$

TOPIC 9

Mensuration: Circles

Introduction

First we review perimeter formulae studied in previous years. We introduce the formula to calculate the length of an arc and see how to use this formula to find the perimeter of a sector of a circle.

Then we deal with the area of plane shapes, and revise formulae learnt in previous years. We introduce a formula to calculate the area of a sector of a circle. Then we use this formula to derive a method of calculating the area of a segment of a circle.

Common difficulties

Students sometimes confuse the different parts of a circle. Make sure that they know the difference between a radius and a diameter, and understand that an arc is part of a circumference. In this section of work, they need to understand the difference between a sector and a segment of a circle.

Students should know the formulae for perimeter and area for quadrilaterals and triangles. They also need to be able to substitute values into the formulae correctly. For example, when calculating the area of the circle, only the radius is squared: Area = πr^2. They must not multiply $\pi \times r$ and then square the answer as that method is incorrect.

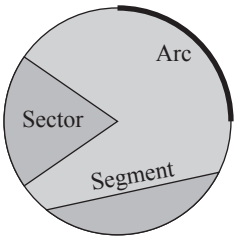
Preparation

Find various objects that have a circular shape or the shape of part of a circle to illustrate the usefulness of this section of work in everyday life, for example, wheels of different sizes. Protractors from students' maths sets can be used to illustrate semicircles.

Have cardboard or paper available from which to cut out various circles, to demonstrate:

- how to find the length of an arc and of the perimeter
- how to measure angles and compare ratios
- how to find the area of segments and sectors.

You can use variations of the circle shown below.



Prepare a chart from a table of all the necessary formulae for perimeter, circumference and area (you will find these in the Student's Book).

Introduction for students

Firstly, discuss the different parts of a circle and introduce the terms sector and segment.

Then revise the formulae for perimeter of a variety of shapes. Discuss the circumference of a circle. Note that the ratio of the circumference of any circle to its diameter is always equal to π. Test this by measuring a few circle shapes.

Explain the term arc and discuss minor and major arcs. Note that a semicircle is an arc with an angle of 180º at the centre of the circle. The arc length is equal to half the circumference of the circle.

Discuss the length of an arc and the proportionality of the length to the size of the angle at the centre. This gives the formula:

Arc length = $\frac{\theta}{360} \times 2\pi r$.

Make sure that students understand the difference between a sector of a circle and a segment of a circle.

If students do not have calculators, tell them that they should use $\pi = 3.14$ in their calculations.

Answers

Exercise 9.1

1. **a)** 26.4 cm **b)** 77.2 cm **c)** 27.9 cm
d) 76.3 cm

2. **a)** 2.9 cm **b)** 7.8 cm **c)** 10.7 cm
d) 8.8 mm

3. **a)** 5.8 cm **b)** 20.7cm **c)** 10.7 cm
d) 13.2 cm

4. **a)** 21.5 cm **b)** 6.8 cm **c)** 6.0 cm
d) 7.4 cm

5. **a)** $\alpha = 36.7^\circ$ **b)** $\alpha = 143.3^\circ$ **c)** $\alpha = 84.3^\circ$
d) $\alpha = 26.4^\circ$

6. **a)** 298 cm **b)** 84.5 cm **c)** 56.6 cm
d) 76.2 cm

7. **a)** 35.9 mm **b)** 38.7 mm **c)** 47.2 mm
d) 58.4 mm

8. 9 420 cm = 94.2 m

9. 45.4 m

10. 21.3 cm

11. 15.4 cm

12. **a)** 29.1 cm
b) 48.8 cm
c) 33.6 cm

Exercise 9.2

1. **a)** 11.0 cm^2 **b)** 180.6 cm^2 **c)** 79.4 cm^2
d) 88.4 cm^2

2. **a)** $r = 12.9$ cm **b)** $r = 12.5$ cm **c)** $r = 16.2$ cm
d) $r = 12.7$ cm

3. **a)** 17.9 cm^2 **b)** 24.1 cm^2

4. **a)** 210.1 cm^2 **b)** 341.6 cm^2 **c)** 147.9 cm^2
d) 523.9 cm^2

5. **a)** 1 045.9 cm^2 **b)** 914.4 cm^2 **c)** 1 108.1 cm^2
d) 732.1 cm^2

6. **a)** 410.4 cm^2 **b)** 207.6 cm^2 **c)** 159.6 cm^2
d) 402.1 cm^2

7. 4 239 mm^2

8. **a)** 21.3 cm^2 **b)** 40 cm^2 **c)** 73.7 cm^2
d) 58.9 cm^2

Assess your progress

1. 78.5 cm^2

2. **a)** 47.1 cm **b)** 62.8°
c) 2 118 cm^2 **d)** 75.9 mm

3. 61.1 cm

4. **a)** 13.4 cm **b)** 107.2 cm^2

5. **a)** 104.5 cm **b)** 565.2 cm^2

6. 4 037.3 mm^2

7. 637 times

8. **a)** 121.9 mm **b)** 837.3 mm^2

9. Perimeter = 33.4 cm
Area = 56.5 cm^2

10. Area of minor segment = 82.4 cm^2. Area of quadrant = 78.5 cm^2. Therefore the area of the minor segment is larger than the area of one quadrant of the circle.

TOPIC 10

Trigonometry

Introduction

In JSS3, your students were introduced to trigonometry. They learnt about the three basic trigonometric ratios and used these ratios to find lengths of sides of right-angled triangles. In SS1, they will build on this knowledge as they use calculators or the trigonometric tables to find the sizes of angles in right-angled triangles. They will derive and apply the trigonometric ratios of 45º, 60º, 30º, 90º and 0º.

They are introduced to angles on the Cartesian plane and use the unit circle to draw graphs of sine and cosine for angles from 0º to 360º. Finally, they solve practical problems involving trigonometry, including problems that involve angles of elevation and depression and compass bearings.

Common difficulties

When solving triangles, units of length need only to be written in the final answer. If no units of length are given, the students should write a number only, or a number followed by 'units'.

When students sketch their own diagrams as an aid to solving a problem, the diagrams do not need to be exactly according to scale, but it will be more helpful if they are at least more or less according to scale. Angle sizes should be more or less realistic. If one given side is longer than another, this should ideally be shown in the diagram as well.

Students need to become comfortable with the trigonometric functions on their calculators. Most importantly, they need to know how to recognise the Degree mode on their own model of calculator, and set it to this mode. If their calculators are not in Degree mode, all their trigonometric calculations will be incorrect. You will need to remediate this individually, for every different model of calculator.

If the students get an error on their calculators when using $\sin^{-1}$ or $\cos^{-1}$ to calculate an angle, it is likely that the value that they are entering is greater than 1 or smaller than –1. This is of course undefined, so the students need to check their work carefully to find the cause of the problem.

When using the trigonometric tables on pages 286 to 297 of the Student's Book, some of your students might find the different tables confusing and will need careful remediation regarding how to use the different tables for different purposes. Check to make sure that they understand how and when to use each table.

Rounding errors are common in trigonometric calculations. Check to see why these are happening. Do your students know how to round off correctly to a given number of decimal places? In a multi-step calculation, are they rounding intermediate answers off? Advise them to round off their final answers only.

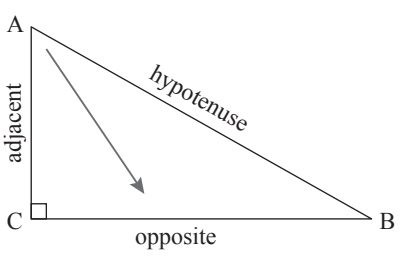
Remind students that in calculations and proofs they must give reasons next to the geometric and trigonometric facts they use. They may put the reasons in brackets, but this is not obligatory.

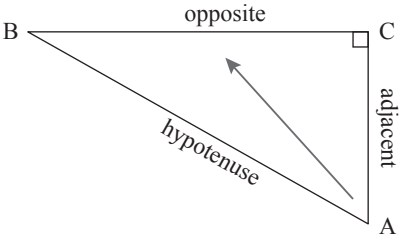
Preparation

Prepare the following charts:

- a chart showing the basic trigonometric ratios, as in the example below:

The trigonometric ratios





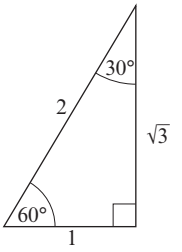
$\text{sine A} = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad \sin A = \dfrac{O}{H}$

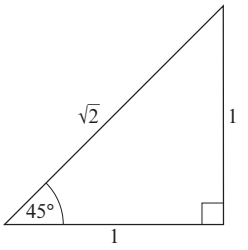
$\text{cosine A} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\quad \cos A = \dfrac{A}{H}$

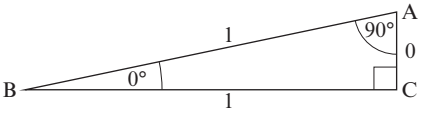
$\text{tangent A} = \dfrac{\text{opposite}}{\text{adjacent}}$ $\quad \tan A = \dfrac{O}{A}$

Use a mnemonic! **S** O H **C** A H **T** O A

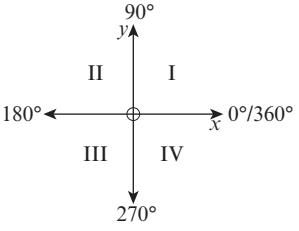
- a chart showing the following special triangles:

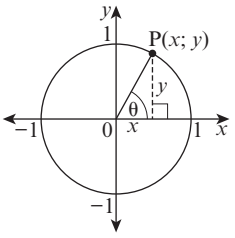






- a chart showing the layout of the Cartesian plane, the unit circle, and the CAST diagram:





Introduction for students

Briefly revise the trigonometry that your students learnt in JSS3. Make sure that they remember and understand the definitions of the three basic trigonometric ratios.

Also revise the theorem of Pythagoras with your class. Draw a few examples of right-angled triangles on the board, where two sides are given. Ask a few volunteers to do the calculations on the board and check that they have remembered when to add, when to subtract, and to take the square root to find the answer. Point out that the hypotenuse is always the longest side in any right-angled triangle.

Answers

Exercise 10.1

1. a) $\sin A = \frac{4}{5}$ **b)** $\cos A = \frac{3}{5}$ **c)** $\tan A = \frac{4}{3}$

d) $\sin B = \frac{3}{5}$ **e)** $\cos B = \frac{4}{5}$ **f)** $\tan B = \frac{3}{4}$

2. a) $\sin D = \frac{77}{85}$ **b)** $\cos D = \frac{36}{85}$ **c)** $\tan D = \frac{77}{36}$

d) $\sin F = \frac{36}{85}$ **e)** $\cos F = \frac{77}{85}$ **f)** $\tan F = \frac{36}{77}$

3. a) $HI^2 = GH^2 + GI^2$ (Pythagoras)

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore HI = \sqrt{169} = 13$

b) $\sin H = \frac{12}{13}$ **c)** $\cos H = \frac{5}{13}$ **d)** $\tan H = \frac{12}{5}$

e) $\sin I = \frac{5}{13}$ **f)** $\cos I = \frac{12}{13}$ **g)** $\tan I = \frac{5}{12}$

4. a) $KL^2 = JL^2 - JK^2$ (Pythagoras)

$= 73^2 - 55^2$

$= 5\,329 - 3\,025$

$= 2\,304$

$\therefore KL = \sqrt{2\,304} = 48$

b) $\sin J = \frac{48}{73}$ **c)** $\cos J = \frac{55}{73}$ **d)** $\tan J = \frac{48}{55}$

e) $\sin L = \frac{55}{73}$ **f)** $\cos L = \frac{48}{73}$ **g)** $\tan L = \frac{55}{48}$

5. a) $MO^2 = MN^2 + NO^2$ (Pythagoras)

$= 2^2 + 3^2$

$= 4 + 9$

$= 13$

$\therefore MO = \sqrt{13}$

b) $\sin M = \frac{3}{\sqrt{13}}$ **c)** $\cos M = \frac{2}{\sqrt{13}}$ **d)** $\tan M = \frac{3}{2}$

e) $\sin O = \frac{2}{\sqrt{13}}$ **f)** $\cos O = \frac{3}{\sqrt{13}}$ **g)** $\tan O = \frac{2}{3}$

Exercise 10.2

1. $\tan 74^\circ = 3.49$ **2.** $\cos 55^\circ = 0.57$ **3.** $\sin 5^\circ = 0.09$

4. $\cos 66^\circ = 0.41$ **5.** $\sin 63^\circ = 0.89$ **6.** $\tan 45^\circ = 1.00$

7. $\sin 21^\circ = 0.36$ **8.** $\cos 77^\circ = 0.22$

Exercise 10.3

1. $\theta = 64.5^\circ$ **2.** $\theta = 30.1^\circ$ **3.** $\theta = 9.8^\circ$

4. $\theta = 83.6^\circ$ **5.** $\theta = 44.8^\circ$ **6.** $\theta = 38.3^\circ$

7. $\theta = 63.4^\circ$ **8.** $\theta = 68.9^\circ$

Exercise 10.4

1. a) $\hat{P} = 180^\circ - 90^\circ - 32^\circ$ (sum of $\angle$s of a $\triangle$)
$= 58^\circ$

b) $\sin 32^\circ = \frac{6.5}{PQ}$

$\therefore PQ \sin 32^\circ = 6.5$

$\therefore PQ = \frac{6.5}{\sin 32^\circ}$

$= 12.27$ cm

c) $\frac{QR}{12.27} = \cos 32^\circ$

$\therefore QR = 12.27 \times \cos 32^\circ$

$= 10.41$ cm

2. a) $\cos X = \frac{8}{17}$

$\hat{X} = 61.9^\circ$

b) $\hat{Z} = 180^\circ - 90^\circ - 61.9^\circ$ (sum of $\angle$s of a $\triangle$)
$= 28.1^\circ$

c) $\frac{YZ}{17} = \sin 61.9^\circ$

$\therefore YZ = 17 \times \sin 61.9^\circ$

$= 15$

d) $YZ^2 = XZ^2 - XY^2$ (Pythagoras)

$= 17^2 - 8^2$

$= 289 - 64$

$= 225$

$\therefore YZ = \sqrt{225}$

$= 15$

3. Students solve each of the given triangles. They may do this in any order and using any method, but they must get the answers below.

a) $\hat{Y} = 180^\circ - 90^\circ - 52^\circ$ (sum of $\angle$s of a $\triangle$)

$= 38^\circ$

$\frac{XY}{3} = \sin 52^\circ$

$\therefore XY = 3 \times \sin 52^\circ$

$= 2.36$ km

$\frac{XZ}{3} = \cos 52^\circ$

$\therefore XZ = 3 \times \cos 52^\circ$

$= 1.85$ km

b) $AB^2 = AC^2 + BC^2$ (Pythagoras)

$= 1 + 4$

$= 5$

$\therefore AB = \sqrt{5}$

$= 2.24$

$\tan B = \frac{1}{2}$

$\therefore \hat{B} = 26.6^\circ$

$\hat{A} = 180^\circ - 90^\circ - 26.6^\circ$ (sum of $\angle$s of a $\triangle$)

$= 63.4^\circ$

c) $DE^2 = DF^2 - EF^2$ (Pythagoras)

$= 13^2 - 5^2$

$= 169 - 25$

$= 144$

$\therefore DE = \sqrt{144}$

$= 12$

$\sin D = \frac{5}{13}$

$\therefore \hat{D} = 22.6^\circ$

$\hat{F} = 180^\circ - 90^\circ - 22.6^\circ$ (sum of $\angle$s of a $\triangle$)

$= 67.4^\circ$

d) $\hat{D} = 180^\circ - 90^\circ - 25^\circ$ (sum of $\angle$s of a $\triangle$)

$= 65^\circ$

$\frac{DE}{3} = \tan 25^\circ$

$\therefore DE = 18 \times \tan 25^\circ$

$= 8.39$

$\cos 25^\circ = \frac{18}{DF}$

$\therefore DF = \frac{18}{\cos 25^\circ}$

$= 19.86$

4. For each of the given triangles, students draw a rough sketch of the triangle and then solve the triangle. They may do this in any order and using any method, but they must get the answers below.

a) $\cos R = \frac{6}{7.5}$

$\therefore \hat{R} = 36.9^\circ$

$\hat{I} = 180^\circ - 90^\circ - 36.9^\circ$ (sum of $\angle$s of a $\triangle$)

$= 53.1^\circ$

$\frac{IT}{6} = \tan 36.9^\circ$

$\therefore IT = 6 \times \tan 36.9^\circ$

$= 4.50$ cm

b) $\hat{L} = 180^\circ - 90^\circ - 5^\circ$ (sum of $\angle$s of a $\triangle$)

$= 85^\circ$

$\frac{AL}{25} = \tan 5^\circ$

$\therefore AL = 25 \times \tan 5^\circ$

$= 2.19$ mm

$\cos 5^\circ = \frac{25}{HL}$

$\therefore HL = \frac{25}{\cos 5^\circ}$

$= 25.10$ mm

c) $\tan G = \frac{31}{58}$

$\therefore \hat{G} = 28.1^\circ$

$\hat{D} = 180^\circ - 90^\circ - 28.1^\circ$ (sum of $\angle$s of a $\triangle$)

$= 61.9^\circ$

$DG^2 = DO^2 + GO^2$ (Pythagoras)

$= 31^2 + 58^2$

$= 4\ 325$

$\therefore DG = \sqrt{4\ 325}$

$= 65.76$

d) $\hat{F} = 180^\circ - 90^\circ - 32^\circ$ (sum of $\angle$s of a $\triangle$)

$= 58^\circ$

$\frac{FL}{83} = \sin 32^\circ$

$\therefore FL = 83 \times \sin 32^\circ$

$= 43.98$

$\frac{LY}{83} = \cos 32^\circ$

$\therefore LY = 83 \times \cos 32^\circ$

$= 70.39$

e) $CT^2 = AC^2 - CT^2$ (Pythagoras)

$= 14^2 - 7^2$

$= 147$

$\therefore CT = \sqrt{147}$

$= 12.12$ m

$\cos A = \frac{7}{14}$

$\therefore \hat{A} = 60^\circ$

$\hat{C} = 180^\circ - 90^\circ - 60^\circ$ (sum of $\angle$s of a $\triangle$)

$= 30^\circ$

f) $\hat{A} = 180^\circ - 90^\circ - 61^\circ$ (sum of $\angle$s of a $\triangle$)

$= 29^\circ$

$\frac{AT}{7.3} = \tan 61^\circ$

$\therefore AT = 7.3 \times \tan 61^\circ$

$= 13.17$ m

$\cos 61^\circ = \frac{7.3}{AN}$

$\therefore AN = \frac{7.3}{\cos 61^\circ}$

$= 15.06$ m

g) $PG^2 = GI^2 - IP^2$ (Pythagoras)

$= 5.75^2 - 3.25^2$

$= 22.5$

$\therefore CT = \sqrt{22.5}$

$= 4.74$

$\sin G = \frac{3.25}{5.75}$

$\therefore \hat{G} = 34.4^\circ$

$\hat{I} = 180^\circ - 90^\circ - 34.4^\circ$ (sum of $\angle$s of a $\triangle$)

$= 55.6^\circ$

h) $\hat{H} = 180^\circ - 90^\circ - 48^\circ$ (sum of $\angle$s of a $\triangle$)

$= 42^\circ$

$\frac{HP}{165} = \tan 48^\circ$

$\therefore HP = 165 \times \tan 48^\circ$

$= 183.25$

$\cos 48^\circ = \frac{165}{HO}$

$\therefore HO = \frac{165}{\cos 48^\circ}$

$= 246.59$

5. a) $\frac{OM}{AM} = \tan A$

$\therefore AM = \frac{OM}{\tan A}$

$= \frac{15}{\tan 36^\circ}$

$= 20.65$ m

$\therefore AB = 2 \times AM$ (OM bisects AB)

$= 2 \times 20.65$ m

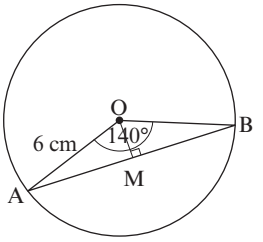
$= 41.30$ m

b) $\hat{A} = \hat{B}$ (isosceles $\triangle$; radii)

$\therefore \hat{A} = \frac{1}{2}(180^\circ - 140^\circ)$ (sum of $\angle$s of a $\triangle$)

$= 20^\circ$

Construct OM $\perp$ AB with M on AB, as shown below:



$\frac{AM}{AO} = \cos A$

$\therefore AM = AO \times \cos A$

$= 6 \times \cos 20^\circ$

$= 5.64$ cm

$\therefore AB = 2 \times AM$ (OM bisects AB)

$= 2 \times 5.64$ cm

$= 11.28$ cm

c) $\frac{AB}{AC} = \cos A$

$\therefore AB = AC \times \cos A$

$= 156 \times \cos 30^\circ$

$= 135.10$ mm

d) First calculate the radius of the circle.

In $\triangle$DNO:

$$\frac{NO}{DO} = \sin D$$

$$\therefore DO = \frac{NO}{\sin D}$$

$$= \frac{2}{\sin 55^\circ}$$

$$= 2.44 \text{ m}$$

In $\triangle$AMO:

$$\frac{AM}{AO} = \cos A$$

$$\therefore AM = AO \times \cos A$$

$$= 2.44 \times \cos 28^\circ$$

$$= 2.15 \text{ m}$$

$$\therefore AB = 2 \times AM \text{ (OM bisects AB)}$$

$$= 2 \times 2.15 \text{ cm}$$

$$= 4.30 \text{ cm}$$

Exercise 10.5

1. The students discuss the three special triangles. Those members of the group who understand these triangles should help the other members of the group to understand them as well.

2. The students close their textbooks. Each student should try to draw these triangles for themselves. They check one another's work and help one another to get it right.

3. **a)** $\sin 0^\circ = 0$ **b)** $\cos 0^\circ = 1$

c) $\tan 0^\circ = 0$ **d)** $\sin 30^\circ = \frac{1}{2}$

e) $\cos 30^\circ = \frac{\sqrt{3}}{2}$ **f)** $\tan 30^\circ = \frac{1}{\sqrt{3}}$

g) $\sin 45^\circ = \frac{1}{\sqrt{2}}$ **h)** $\cos 45^\circ = \frac{1}{\sqrt{2}}$

i) $\tan 45^\circ = 1$ **j)** $\sin 60^\circ = \frac{\sqrt{3}}{2}$

k) $\cos 60^\circ = \frac{1}{2}$ **l)** $\tan 60^\circ = \sqrt{3}$

m) $\sin 90^\circ = 1$ **n)** $\cos 90^\circ = 0$

o) $\tan 90^\circ$ = undefined

4. a) The values of the sine of an angle increase from 0 to 1 as the angle increases from 0º to 90º.

b) The values of the cosine of an angle decrease from 1 to 0 as the angle increases from 0º to 90º.

c) The values of the tangent of an angle increase from 0 as the angle increases from 0º to 90º.

d) Tan 90º is undefined.

5. – 7. Student's own work and group work.

Exercise 10.6

1. $\cos 60^\circ - \sin 30^\circ = \frac{1}{2} - \frac{1}{2} = 0$

2. $\sin^2 45^\circ + \cos^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

3. $(\sin 45^\circ + \cos 45^\circ)^2 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = 2$

4. $\tan^2 60^\circ + \tan^2 45^\circ + \tan^2 30^\circ = \left(\frac{\sqrt{3}}{1}\right)^2 + (1)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + 1 + \frac{1}{3}$

$= 4\frac{1}{3}$

5. $\frac{\sin 60^\circ}{\sin 45^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{6}}{2}$

6. $\left(\frac{1}{\cos 30^\circ}\right)\left(\frac{1}{\cos 45^\circ}\right)\left(\frac{1}{\cos 60^\circ}\right) = \left(\frac{2}{\sqrt{3}}\right)\left(\frac{\sqrt{2}}{1}\right)\left(\frac{2}{1}\right) = 4\sqrt{\frac{2}{3}}$

7. $2\cos 30^\circ \sin 30^\circ + \sin 60^\circ \sin 90^\circ = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)(1)$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

8. $\frac{\cos 30^\circ \tan 45^\circ \sin 30^\circ}{\sin 60^\circ} = \frac{\left(\frac{\sqrt{3}}{2}\right)(1)\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}} = \frac{1}{2}$

9. $\frac{\sin 45^\circ \tan^2 60^\circ}{\cos 45^\circ} = \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{1}\right)^2}{\frac{1}{\sqrt{2}}} = 3$

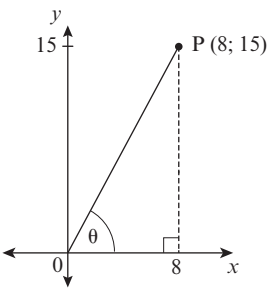
10. $\tan^2 60^\circ - 2\sin 30^\circ \cos 60^\circ + 2\cos^2 45^\circ \cos 0^\circ$

$= \left(\frac{\sqrt{3}}{1}\right)^2 - 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + 2\left(\frac{1}{\sqrt{2}}\right)^2(1)$

$= 3\frac{1}{2}$

Exercise 10.7

1. a)



$OP^2 = 8^2 + 15^2$ (Pythagoras)

$= 64 + 225$

$= 289$

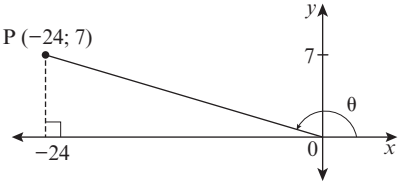
$\therefore OP = \sqrt{289} = 17$

(i) $\sin\theta = \frac{15}{17}$

(ii) $\cos\theta = \frac{8}{17}$

(iii) $\tan\theta = \frac{15}{8}$

b)



$OP^2 = (-24)^2 + 7^2$ (Pythagoras)

$= 576 + 49$

$= 625$

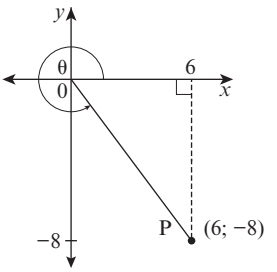
$\therefore OP = \sqrt{625} = 25$

(i) $\sin\theta = \frac{7}{25}$

(ii) $\cos\theta = -\frac{24}{25}$

(iii) $\tan\theta = -\frac{7}{24}$

c)



$OP^2 = 6^2 + (-8)^2$ (Pythagoras)

$= 36 + 64$

$= 100$

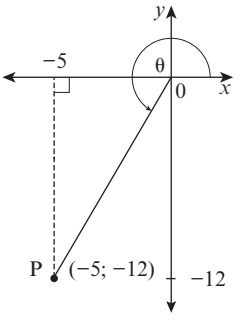
$\therefore OP = \sqrt{100} = 10$

(i) $\sin\theta = -\frac{8}{10} = -\frac{4}{5}$

(ii) $\cos\theta = \frac{6}{10} = \frac{3}{5}$

(iii) $\tan\theta = -\frac{8}{6} = -\frac{4}{3}$

d)



$OP^2 = (-5)^2 + (-12)^2$ (Pythagoras)

$= 25 + 144$

$= 169$

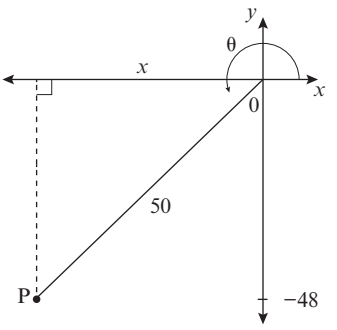
$\therefore OP = \sqrt{169} = 13$

(i) $\sin\theta = -\frac{12}{13}$

(ii) $\cos\theta = -\frac{5}{13}$

(iii) $\tan\theta = \frac{-12}{-5} = \frac{12}{5}$

2. a)



b) $x^2 = 50^2 - (-48)^2$ (Pythagoras)

$= 2\,500 - 2\,304$

$= 196$

$\therefore x = -\sqrt{196} = -14$

(i) $\cos\theta = -\frac{14}{50} = -\frac{7}{25}$

(ii) $\tan\theta = \frac{-48}{-14} = \frac{24}{7}$

c) $\frac{\sin\theta}{\cos\theta} = -\frac{48}{50} \div -\frac{14}{50}$

$= -\frac{48}{50} \times -\frac{50}{14}$

$= \frac{48}{14}$

$= \frac{24}{7}$

$= \tan\theta$

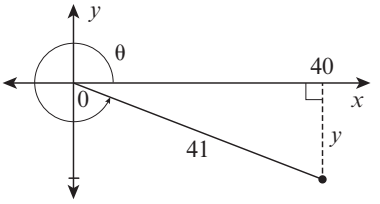
d) $\sin^2\theta + \cos^2\theta = \left(-\frac{48}{50}\right)^2 + \left(-\frac{14}{50}\right)^2$

$= \frac{2\,304}{2\,500} + \frac{196}{2\,500}$

$= \frac{2\,500}{2\,500}$

$= 1$

3.



$y^2 = 41^2 - 40^2$ (Pythagoras)

$= 1\,681 - 1\,600$

$= 81$

$\therefore y = -\sqrt{81} = -9$

$\tan\theta = -\frac{9}{40}$

$\sin\theta = -\frac{9}{41}$

Exercise 10.8

1. Students will need a pair of compasses and a protractor.

a) Students draw a system of axes and construct a circle with the midpoint on the origin and a radius of 10 cm.

b) Students divide each quadrant of the Cartesian plane into intervals of 30º.

c)

Angle (θ)	0º	30º	60º	90º	120º	150º	180º	210º	240º	270º	300º	330º	360º
cos θ (x coordinate)	1	0.87	0.5	0	–0.5	–0.87	–1	–0.87	–0.5	0	0.5	0.87	1
sin θ (y coordinate)	0	0.5	0.87	1	0.87	0.5	0	–0.5	–0.87	–1	–0.87	–0.5	0

2. Students draw the graph of $y = \sin x$ for $0º \leq x \leq 360º$, using intervals of 30º. Use graph in Student's Book as a guide.

3. Students draw the graph of $y = \cos x$ for $0º \leq x \leq 360º$, using intervals of 30º. Use graph in Student's Book as a guide.

4. a) –1 **b)** 1 **c)** –1
d) 1

Exercise 10.9

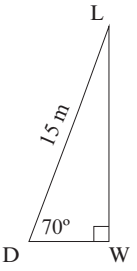
1. a) 0 **b)** 1 **c)** 0
d) –1 **e)** 0 **f)** 1
g) 0 **h)** –1 **i)** 0
j) 1

2. a) 0º, 180º, 360º **b)** 90º **c)** 270º
d) 90º, 270º **e)** 0º, 360º **f)** 180º

3. a) 0º and 180º **b)** 180º and 360º **c)** 90º and 270º

Exercise 10.10

1. a)



b) $\frac{LW}{LD} = \sin D$

$\therefore LW = 15 \times \sin 70^\circ$

$= 14.10 \text{ m}$

c) $\frac{DW}{LD} = \cos D$

$\therefore DW = 15 \times \cos 70^\circ$

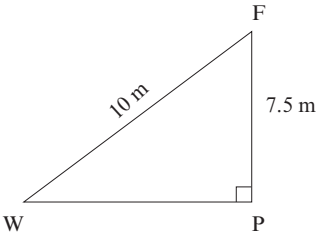
$= 5.13 \text{ m}$

2. Let the angle be θ.

$\tan \theta = \frac{2}{50}$

$\therefore \theta = 2.3^\circ$

3. a)



b) $\sin W = \frac{7.5}{10}$

$\therefore \hat{W} = 48.60^\circ$

c) $WP^2 = 10^2 - 7.5^2$ (Pythagoras)

$= 100 - 56.25$

$= 43.75$

$\therefore WP = \sqrt{43.75}$

$= 6.61 \text{ m}$

4. $\frac{XY}{YZ} = \tan Z$

$\therefore \frac{XY}{61} = \tan 30^\circ$

$\therefore XY = 61 \times \tan 30^\circ$

$= 35.22 \text{ m}$

Exercise 10.11

1. $\frac{TR}{BR} = \tan B$

$\therefore TR = BR \times \tan B$

$= 15 \times \tan 29^\circ$

$= 8.31 \text{ m}$

2. $\frac{BP}{BF} = \tan F$

$\therefore BP = BF \times \tan F$

$= 10 \times \tan 4^\circ$

$= 0.699 \text{ km}$

$= 699 \text{ m}$

3. a) Point B represents the spot from which the angle of elevation was 31.8º.
Point C represents the foot of the cliff.
Point D represents the eagle.

b) In $\triangle ACD$:

$\frac{DC}{AC} = \tan A$

$\therefore DC = AC \times \tan A$

$= (500 + x) \times \tan 22.3^\circ$

$= (500 + x) \times 0.41$

$= 205 + 0.41x$ ①

In $\triangle BCD$:

$\frac{DC}{BC} = \tan B$

$\therefore DC = BC \times \tan B$

$= x \times \tan 31.8^\circ$

$= x \times 0.62$

$= 0.62x$ ②

Substitute ② into ①:

$$0.62x = 205 + 0.41x$$

$$\therefore 0.62x - 0.41x = 205$$

$$\therefore 0.21x = 205$$

$$\therefore x = \frac{205}{0.21}$$

$$= 976 \text{ m}$$

c) $DC = 0.62x$

$= 0.62 \times 976$

$= 605$ m

4. a) $\frac{MS}{CS} = \tan M\hat{C}S$

$\therefore MS = CS \times \tan M\hat{C}S = 20 \text{ m} \times \tan 45^\circ = 20$ m

b) $D\hat{M}B = M\hat{B}S$ (alt. ∠s; DM || BS)

$M\hat{B}S = \tan^{-1} \frac{20}{30} = 33.7^\circ$

$\therefore D\hat{M}B = 33.7^\circ$

c) $M\hat{B}S = \tan^{-1} \frac{20}{40} = 26.6^\circ$

Exercise 10.12

Students do this Exercise as a group activity.

1. a) N **b)** W **c)** SE

2. a) 180º **b)** 045º **c)** 225º

Exercise 10.13

1. a) $HM = HW = \text{speed} \times \text{time} = 4.5 \text{ km/h} \times \frac{1}{3} \text{ h} = 1.5$ km

$M\hat{H}W = 135^\circ - 45^\circ = 90^\circ$

$\hat{M} = \hat{W} = 45^\circ$ (isosceles △HMW)

$\sin M = \frac{HW}{MW}$

$\therefore \sin 45^\circ = \frac{1.5}{MW}$

$\therefore \frac{1}{\sqrt{2}} = \frac{1.5}{MW}$

$\therefore MW = 1.5 \times \sqrt{2} = 2.12$ km

Each of them jogged half of MW, which is 1.06 km.

Time to jog 1.06 km $= \frac{\text{distance}}{\text{speed}}$

$= \frac{1.06 \text{ km}}{4.5 \text{ km/h}}$

$= 0.235$ hours

$= 14.13$ minutes.

So each of them jogged for $20 + 14 = 34$ minutes.

b) Each of them jogged for $1.5 + 1.06 = 2.56$ km.

2. a) $A\hat{B}N = 180^\circ - 70^\circ$ (coint. ∠s; North lines ||)

$= 110^\circ$

$\therefore A\hat{B}C = 360^\circ - 210^\circ - 110^\circ$ (∠s around a point)

$= 40^\circ$

b) $N\hat{C}B = 180^\circ - 110^\circ - 40^\circ$ (coint. ∠s; North lines ||)

$= 30^\circ$

$N\hat{C}A = 360^\circ - 340^\circ$ (∠s around a point)

$= 20^\circ$

$\therefore A\hat{C}B = 30^\circ + 20^\circ = 50^\circ$

c) $B\hat{A}C = 180^\circ - 40^\circ - 50^\circ$ (sum of ∠s of a △)

$= 90^\circ$

d) $\cos B = \frac{AB}{BC}$

$\therefore \cos 40^\circ = \frac{150}{BC}$

$\therefore BC = \frac{150}{\cos 40^\circ}$

$= 195.81$ km

e) $\tan B = \frac{AC}{AB}$

$\therefore \tan 40^\circ = \frac{AC}{150}$

$\therefore AC = 150 \times \tan 40^\circ$

$= 125.86$ km

3. a) $A\hat{J}N = 360^\circ - 330^\circ$ ($\angle$s around a point)

$= 30^\circ$

$B\hat{A}J = 180^\circ - 60^\circ - 30^\circ$ (coint. $\angle$s; North lines ||)

$= 90^\circ$

$\therefore \tan A\hat{J}B = \frac{AB}{AJ} = \frac{40}{30}$

$\therefore A\hat{J}B = \tan^{-1}\left(\frac{40}{30}\right) = 53.1^\circ$

$\therefore N\hat{J}B = 53.1^\circ - 30^\circ = 023.1^\circ$

b) $BJ^2 = AB^2 + AJ^2$ (Pythagoras)

$= 40^2 + 30^2 = 2\,500$

$\therefore BJ = \sqrt{2\,500} = 50$ km

Assess your progress

1. a) $KL^2 = JK^2 - JL^2$ (Pythagoras)

$= 29^2 - 21^2 = 841$

$441 = 400$

$\therefore KL = \sqrt{400} = 20$

b) $\cos J = \frac{21}{29}$ **c)** $\tan K = \frac{21}{20}$ **d)** $\sin J = \frac{20}{29}$

2. Students draw a rough sketch of $\triangle ABC$ and then solve the triangle. They may do this in any order and using any method, but they must get the answers below.

$AC^2 = 5^2 + 8^2$ (Pythagoras)

$= 89$

$\therefore AC = \sqrt{89}$

$= 9.43$ cm

$\tan C = \frac{5}{8}$

$\therefore \hat{C} = 32.0^\circ$

$\hat{A} = 180^\circ - 90^\circ - 32.0^\circ$ (sum of $\angle$s of a $\triangle$)

$= 58.0^\circ$

3. a) $\frac{MQ}{OQ} = \cos Q$

$\therefore MQ = OQ \times \cos A$

$= 7 \times \cos 30^\circ$

$= 6.06$ cm

$\therefore PQ = 2 \times MQ$ (OM bisects PQ)

$= 2 \times 6.06$ cm

$= 12.12$ cm

b) $RQ = 2 \times 2$ m $= 4$ m

$\frac{PQ}{RQ} = \sin R$

$\therefore PQ = RQ \times \sin R$

$= 4 \times \sin 69^\circ$

$= 3.73$ m

4. a) $\cos 45^\circ - \sin 45^\circ = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

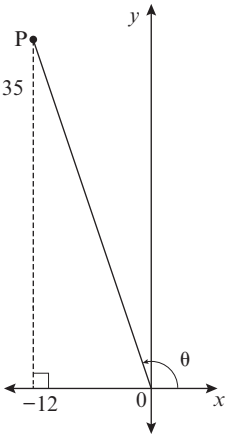
b) $\sin 30^\circ \tan 45^\circ - \sin^2 45^\circ = \left(\frac{1}{2}\right)(1) - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} - \frac{1}{2} = 0$

c) $\frac{\cos 30^\circ}{\tan 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{1}} = \frac{1}{2}$

d) $3 \tan^2 60^\circ + 2 \sin^2 45^\circ + 4 \cos^2 90^\circ$

$= 3\left(\frac{\sqrt{3}}{1}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 4(0)^2 = 10$

5. a)



b) $OP^2 = 35^2 + (-12)^2$ (Pythagoras)

$= 1\ 369$

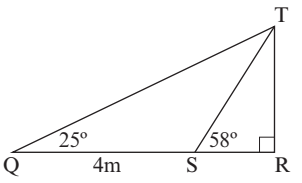
$\therefore OP = \sqrt{1\ 369} = 37$

$\therefore \sin\theta = \frac{35}{37}$ and $\cos\theta = -\frac{12}{37}$

6. The amplitude of the sine graph is 1. This is half the distance between the highest and the lowest points of the graph.

7. The period of the cosine graph is 360º. This is the interval on the x axis over which the graph completes one full cycle.

8. a)



b) Let SR = x.

$\frac{TR}{x} = \tan 58º$

$\therefore TR = x \times \tan 58º$ ①

$\frac{TR}{x+4} = \tan 25º$

$\therefore TR = (x + 4) \times \tan 25º$ ②

Substitute ② into ①:

$(x + 4)\tan 25º = x \times \tan 58º$

$\therefore x(\tan 58º - \tan 25º) = 4 \times \tan 25º$

$\therefore x = \frac{4 \tan 25º}{\tan 58º - \tan 25º}$

$\therefore x = 1.645$ m ③

Substitute ③ into ①:

$TR = 1.645 \times \tan 58º$

$= 2.63$ m

9. a) $Q\hat{P}R = 180^o - 145^o$ ($\angle$s on a straight line)

$= 35^o$

$\tan Q\hat{P}R = \frac{QR}{PQ}$

$\therefore PQ = \frac{75}{\tan 35^o} = 107.11$ m

b) $P\hat{R}Q = 180^o - 90^o - 35^o$ (sum of $\angle$s of a $\triangle$)

$\cos P\hat{R}Q = \frac{QR}{PR}$

$\therefore PR = \frac{75}{\cos 55^o} = 130.76$ m

c) 000^o

TOPIC 11

Logical reasoning

Introduction

In this topic, your students are introduced to logical reasoning for the first time. This is a fascinating field of study that is applied in many diverse fields that range from computer logic to philosophy.

First your students will learn how to identify a simple statement and to distinguish between open and closed statements. They will also learn how to formulate the negation of a simple statement.

The next section deals with compound statements and the five logical operations, as well as truth tables for these operations.

Common difficulties

As is always the case with any new work, some of your students will enjoy being exposed to new ideas, while others will resist the novelty and unfamiliarity of the unknown. Be patient with these students and allow them to ask any questions that they might have. Also allow them to do as many examples as possible.

Most students should cope fairly well with simple statements, but many will find the idea of the different logical operations in compound statements confusing. Spend as much time as necessary on the different truth tables and make sure that your students all understand how to interpret these tables.

The truth table for conditional statements can be particularly tricky to understand. You can reproduce it as a chart for your classroom:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Many of your students will find it strange that if P is false, $P \Rightarrow Q$ is always true. You could explain this to your class as follows.

View $P \Rightarrow Q$ as a promise that is made:

- If P is true, then I promise that Q will also be true.
- So if P is true and Q is true, the promise has been kept and $P \Rightarrow Q$ is true.
- Likewise, if P is true and Q is false, the promise has been broken and $P \Rightarrow Q$ is false.
- On the other hand, if P is false, the promise is void and cannot be broken.
- So if P is false, $P \Rightarrow Q$ is always true.

Preparation

Prepare the following charts and display them in your classroom:

- examples of simple statements, true and false statements and negation of statements
- truth tables for the five logical operations (negation, conjunction, disjunction, conditional (implication) and bi-conditional (bi-implication)
- examples of compound statements involving conjunction, disjunction, conditional and bi-conditional.

Introduction for students

Write the words ‘logical’ and ‘reasoning’ on the board. Space them out so that each word can be discussed separately.

Have a short class discussion about the word ‘logical’. Ask your class questions like:

- What do you think the word ‘logical’ means?
- Give an example of how one can use the word ‘logical’ in real life.
- Do you think that the word ‘logical’ applies to Mathematics? Why do you say so?

Write any meaningful answers on the board and allow your students to take notes if they wish to do so.

Now repeat the class discussion and the questions, but this time about the word ‘reasoning’.

Explain to your class that in this topic they will learn about the basic components and ideas that we use in logical reasoning. They will learn more about logical reasoning in SS2.

Answers

Exercise 11.1

1. **a)** statement
b) not a statement (a question)
c) statement
d) statement
e) statement
f) statement
g) not a statement (not a complete sentence)
h) statement
i) not a statement (a command)
j) statement

2. a, e and h

3. c and f

4. d and j

Exercise 11.2

1.

	Statement	Open?	Closed	
			True?	False?
a)	All cats are mammals.		✓	
b)	All mammals are cats.			✓
c)	My uncle studies mammals.	✓		
d)	Some cars are silver.		✓	
e)	My father's car is silver.	✓		
f)	All cars are silver.			✓
g)	The number of students in our class is a prime number.	✓		
h)	25 is a prime number.			✓
i)	There is only one even prime number.		✓	

2. a) (i) $x = 2$
(ii) x = any value other than 2
b) (i) $p = -7$
(ii) p = any value other than -7
c) (i) $a > 9$
(ii) $a \leq 9$
d) (i) Student's own answers, for example: $x = 9$ and $y = 3$.
(ii) Student's own answers such that $y \neq \sqrt{x}$.
e) (i) $x = 0$
(ii) x = any value other than 0
f) (i) Student's own answers, for example: $x = 1$ and $y = 5$.
(ii) Student's own answers such that $y \neq x + 4$.
g) (i) Student's own answers, for example: $x = 1$ and $y = -8$.
(ii) Student's own answers such that $y \neq 3x - 11$.
h) (i) $x = 1$
(ii) x = any value other than 1

Exercise 11.3

1. a) My best friend's name is not Abeo.
b) Benin does not share a border with Nigeria.
c) $17 - 5 = 12$
d) Lake Chad is not completely inside Nigeria.
e) $\mathbb{N} \not\subset \mathbb{Q}$
f) $-30 > -31$

2. a) No students enjoy sport.
b) Some adults do not have bank accounts.
c) All dogs chase cats.
d) Some children like spinach.

3. a) (ii), (iii) and (iv)
b) (i) and (iv)
c) (ii) and (iv)
d) (iii) and (iv)

Exercise 11.4

1. a) This is a conjunction.
b) P ∧ Q is true if P and Q are true.
c) P ∧ Q is false if one or both of P and Q are false.

2. a) T ∧ F = F **b)** F ∧ F = F **c)** T ∧ F = F
d) T ∧ T = T **e)** F ∧ T = F **f)** T ∧ T = T

3. a) $x = 5$ and $y = 4$. Note that both statements must be true.
b) There are many possible answers. Here is one solution: $x = 4$ and $y = 5$. Note that at least one of the statements must be false.

Exercise 11.5

1. a) This is a disjunction.
b) P ∨ Q is true if P and/or Q are true.
c) P ∨ Q is false if both P and Q are false.

2. a) F ∨ T = T **b)** F ∨ F = F **c)** T ∨ T = T
d) F ∨ T = T **e)** T ∨ F = T **f)** F ∨ F = F

3. Student's own answers. Below are possible solutions.
a) $x = 9$ and $y = 2$. Note that at least one of the statements must be true.
b) $x = 2$ and $y = 8$. Note that both statements must be false.

Exercise 11.6

1. a) This is a conditional statement.
b) P ⇒ Q is true if P is false or if both P and Q are true.
c) P ⇒ Q is false if P is true and Q is false.

2. a) She saves her money.
b) She will be able to buy a new bicycle.

3. a) T ⇒ T = T **b)** T ⇒ F = F **c)** T ⇒ T = T
d) F ⇒ T = T **e)** F ⇒ F = T **f)** T ⇒ F = F

Exercise 11.7

1. a) This is a bi-conditional statement.
b) P ⇔ Q is true if P and Q have the same truth values.
c) P ⇔ Q is false if P and Q have different truth values.

2. a) T ⇔ T = T **b)** T ⇔ F = F **c)** T ⇔ T = T
d) F ⇔ F = T **e)** F ⇔ T = F **f)** T ⇔ F = F

Exercise 11.8

1. a) not **b)** and **c)** or
d) if … then **e)** if and only if

2.

P	~P
T	F
F	T

3. **a)** It is not cold.
b) The wind is not blowing.
c) It is cold and the wind is blowing.
d) It is cold or the wind is blowing.
e) The wind is blowing.
f) If it is cold, then the wind is blowing.
g) The wind is blowing if and only if it is cold.
h) If it is cold, then the wind is not blowing.

4. **a)** $\sim P$ **b)** $\sim Q$ **c)** $P \wedge Q$
d) $P \wedge \sim Q$ **e)** $Q \wedge \sim P$ **f)** $\sim P \wedge \sim Q$
g) $P \vee Q$ **h)** $P \Rightarrow Q$ **i)** $Q \Rightarrow \sim P$
j) $P \Leftrightarrow Q$

5. **a)** If F then T = T **b)** F if and only if F = T
c) F or T = T **d)** T and F = F
e) If T then F = F

Assess your progress

1. **a)** Open statement **b)** Closed statement
c) Open statement **d)** Not a statement
e) Closed statement **f)** Closed statement

2. **a)** $3 + 12 \neq 16$
b) 15 is not an even number.
c) Nigeria does not share a border with four other countries.
d) 17 is a prime factor of 34.
e) Some boys do not love playing football.
f) All women enjoy baking.

3. *x* represents $\Leftrightarrow$.

4. **a)** $T \wedge F = F$ **b)** $T \vee F = T$ **c)** $T \Rightarrow F = F$
d) $F \Rightarrow T = T$ **e)** $T \wedge \sim F = T$ **f)** $T \Leftrightarrow F = F$

5. **a)** The plural of mouse is mice and the plural of house is hice.
b) The plural of mouse is mice or the plural of house is hice.
c) If the plural of mouse is mice then the plural of house is hice.
d) If the plural of house is hice then the plural of mouse is mice.
e) The plural of mouse is mice and the plural of house is not hice.
f) The plural of mouse is mice if and only if the plural of house is hice.

TOPIC 12

Mensuration: Solids

Introduction

We begin this topic by revising three-dimensional objects already studied in previous years. The solids to be covered are cubes, cuboids, pyramids, cylinders, cones, and prisms.

Then we revise volume, which is the amount of space occupied by a solid, and use the volume formulae to calculate volume of solids or combinations of solids.

We move on to frustrums. A frustrum is the part of a cone or pyramid that is left after its upper part has been cut off by a plane parallel to its base. In this section, students will learn how to calculate the curved surface area of a truncated cone as well as its volume. They also learn how to calculate surface area and volume of a truncated pyramid. Two methods of calculating area and volume of frustrums are shown and either method may be used.

Common difficulties

Students need to know the difference between faces, edges and vertices in three-dimensional objects. They need to be able to recognise and name the different shapes being studied.

Students need to learn all the formulae for surface area and volume and know which formula to use for which shape.

They need to read the questions carefully and decide whether they are being asked to calculate surface area or volume. Then they need to use the correct formula.

It is important that they ensure that all measurements are using the same units. They also need to be able to convert units so that they are always working with the same unit of measurement.

Preparation

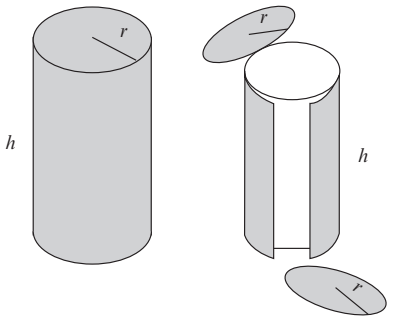
Have a chart with the different solids displayed. Prepare a table of all the necessary formulae for surface area and volume.

Find various objects that have interesting shapes, for example, various cans or boxes of different sizes from a grocery

store, to illustrate the usefulness of this section of work in everyday life.

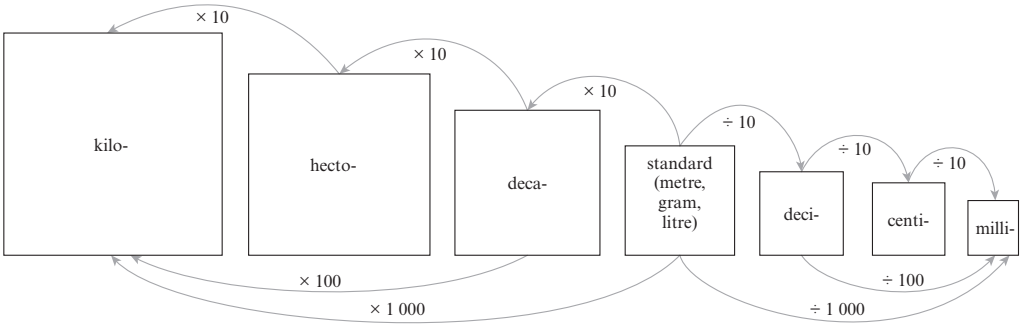
Take some of these solids apart to see the nets of how three-dimensional objects are formed from a two-dimensional net. For example, the cylinder below is made up of two circles and a rectangle which shows how the total surface area formula for a cylinder is made up.

Create models that show the frustrums of a cone and a pyramid.



You could make a chart to show how to convert metric units from one unit to another, and of the symbols and abbreviations students will need to use.

Symbols and abbreviations	
P = perimeter	s = side (square) or slant height
C = circumference	r = radius
A = area	d = diameter
l = length	π = pi, approximately 3.14
b = breadth or base	V = volume
h = vertical height	TSA = Total surface area



Introduction for students

Work through a few examples with the students on converting one metric unit to another.

Explain that a solid is another name for a three-dimensional shape. Remind students that 3D shapes have faces and edges and vertices.

Remind them also that total surface area (TSA) is the sum of the area of all faces or curved surfaces of an object. Area is measured in square units, for example m^2.

Revise surface area and volume with the students. Make sure they know how to use the different formulae as given in the Student's Book. Volume is the amount of space occupied by a solid and is measured in cubic units, for example cm^3.

Explain that you will be studying a new section of work on frustrums. In order to be well prepared to calculate surface area and volume of frustrums, it is very important that the students know their formulae for the surface area and volume of cones and pyramids thoroughly.

For extension purposes, the formulae for curved surface area and volume of a frustrum are shown, but students can work out area and volume of frustrums using their existing knowledge without using these formulae. The worked example shows both methods.

When calculating area and volume involving circles, students may use $\pi = \frac{22}{7}$ or $\pi = 3.14$.

Answers

Exercise 12.1

1. a) 900 mm^2 **b)** 179.02 cm^2 **c)** 61.34 cm^2
d) 11.25 m^2

2. a) 3 480 mm^2 **b)** 696 cm^2 **c)** 2 256 cm^2
d) 12.4 m^2

3. a) 1 024 cm^2 **b)** 9 600 cm^2 **c)** 360 cm^2
d) 3 200 mm^2

4. 1 140 cm^2 **5.** 1 060 cm^2 **6.** 20 cm

7. 3.5 cm **8.** 3 cm **9.** Two

10. 317 m^2

Exercise 12.2

1. a) 282.6 cm^2 **b)** 585 cm^2 **c)** 1 055 mm^2
d) 1 919.2 cm^2

2. a) 678.2 cm^2 **b)** 703.4 mm^2 **c)** 486.7 cm^2
d) 614.6 cm^2

3. 4 158 mm^2

4. 11 657.25 mm^2

5. 477 m^2

Exercise 12.3

1. a) 1 800 mm^2 (or 1.8 cm^2) **b)** 136.7 cm^2
c) 30.7 cm^2 **d)** 2.25 m^2

2. a) 5 120 mm^3 (or 5.12 cm^3) **b)** 57.2 cm^3
c) 146.8 cm^3 **d)** 12.48 m^3

3. a) 10 240 mm^3 **b)** 48 000 mm^3
c) 400 cm^3 **d)** 1 568 cm^3

4. a) 29 920 cm^3
b) 135 cm^3

5. 4 cm^3

6. 1 815.53 cm^3

7. a) 6 667 920 mm^3 (6 667.92 cm^3)
b) 1.664 m^3

8. height = 1.6 m

9. 161 280 m^3

10. 7 967.4 cm^3

Exercise 12.4

1. a) 785 cm^3 **b)** 642.5 m^3 **c)** 1 306.24 mm^3
d) 2 639.17 cm^3

2. a) 1 230.9 mm^3
b) 1 356.48 cm^3
c) 2 512 mm^3

3. 2.4 cm

4. 45.3 cm^3

5. 17 758.79 mm^3

6. 770.3 m^3

7. 628 cm^3

Exercise 12.5

1. a) 241.5 cm^2 **b)** 169.1 cm^2 **c)** 247.4 cm^2

2. a) 7.5 cm **b)** 854.08 cm^2

3. a) 8 cm
b) 384 cm^2
c) $384 + 9 - 4(1.5 \times 2.5) = 378$ cm^2

4. $1\,120.8 - 18^2 = 796.8$ cm^2

5. 596.9 cm^2

6. 710.2 cm^2

Exercise 12.6

1. a) 527.52 cm^3 **b)** 586.13 cm^3 **c)** 1 168.08 cm^3

2. a) 3.6 cm **b)** 21.65 cm^3 **c)** 88.68 cm^3
d) 67.03 cm^3

3. a) 7.5 cm **b)** 125.6 cm^3 **c)** 3 265.6 cm^3

4. a) 8 cm **b)** 378 cm^3

5. 1 864.4 cm^3

6. 2 505.6 cm^3

7. 29 541.1 cm^3

8. **a)** 50 cm **b)** 10 257.3 cm^3 **c)** 82 058.7 cm^3
d) 71 801.4 cm^3 **e)** 30 772 cm^3 **f)** 41 029.4 cm^3

Assess your progress

1. V = 3.375 cm^3 and TSA = 13.5 cm^2

2. a – g, b – h, c – i, d – j, e – f.

3. V = 174 cm^3
TSA = 12 + 24 + 30 + 18 + 2(15) + 2(14) + 42 + 42 = 226 cm^2

4. V = 1 247.1 cm^3

5. TSA = 872.3 cm^2
Volume = 1 706.9 cm^3

6. TSA = 408.2 cm^2
Volume = 628 cm^3

7. 11 124 cm^2

8. **a)** 602.88 cm^3 **b)** 2 304 cm^3 **c)** 1 701.12 cm^3

9. **a)** 751.7 cm^2 **b)** 3 406.12 cm^3

10. 700 cm^3

11. 40.3 cm

TOPIC 13

Constructions

Introduction

Your students have worked with geometrical constructions throughout Junior Secondary. In this topic, your students will revise their knowledge of constructions. They will also work with the locus of a point for the first time.

First your students construct and bisect line segments and angles. In particular, they will construct angles of 90º and 60º, construct and bisect angles to get angles of 30º, 45º and 22.5º and use composite angles to construct angles of 120º, 150º and 135º.

Then they will construct triangles and quadrilaterals from given information and to given specifications.

Finally they construct and describe loci of moving points.

Common difficulties

If your students' constructions are not neat and accurate, consider the following:

- Make sure that their rulers have legible markings.
- Make sure that they are working with sharp pencils.
- Check that their compasses are not loose, and if they are, help them to tighten them with a small screwdriver.
- If students struggle to draw circles and arcs neatly, advise them to try holding the compasses steady and rotate the paper instead of the compasses.

If students struggle to visualise loci, give them more practice with simple loci. Encourage an intuitive approach rather than having your students trying to over-think any given locus.

Where students are to do exercises in pairs or groups, facilitate these interactions.

Preparation

Prepare the following charts to display in your classroom:

- examples of some basic constructions that your students have already done, for example a construction for bisecting a line segment and for bisecting an angle (acute and obtuse)

- the properties of different kinds of triangles categorised by sides (equilateral, isosceles and scalene) as well as by angles (acute-angled, right-angled and obtuse-angled)
- the properties of different special quadrilaterals (a square, a rectangle, a rhombus, a parallelogram, a kite and a trapezium).

You could have cardboard or paper available from which to cut out triangles, and to make models in the shapes of the relevant solids.

Introduction for students

Draw a line segment on the board. Ask your students if they remember how to use a geometrical construction to bisect the line segment.

Now draw an acute angle and an obtuse angle on the board. Ask your class if they remember how to use geometrical constructions to bisect these angles.

Once you have revised these kinds of constructions with your class, have a brief class discussion about the properties of different triangles and quadrilaterals. Point out the charts that you have prepared and advise your students to refer to these charts whenever necessary.

Answers

Note that in the solutions provided, constructions are not drawn to size. The solutions give an idea of what the students' constructions should look like.

Exercise 13.1

Student's own work, in pairs. Ensure students are carrying out the processes correctly.

Exercise 13.2

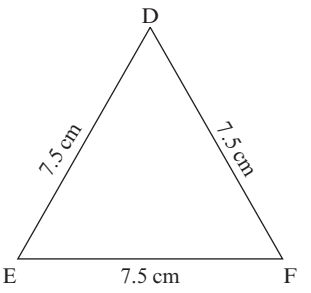
Student's own work, in pairs. Ensure students are carrying out the processes correctly.

Exercise 13.3

Student's own work, in pairs. Ensure students are carrying out the processes correctly.

Exercise 13.4

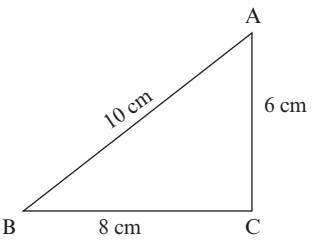
1. a)



b) $\hat{D} = 60°$, $\hat{E} = 60°$ and $\hat{F} = 60°$.

c) $\triangle DEF$ is an equilateral triangle.

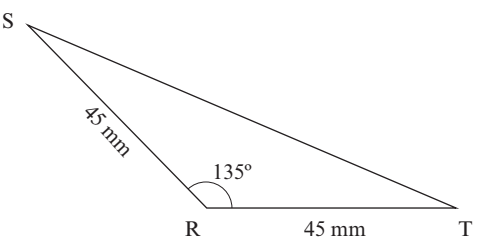
2. a)



b) $\hat{A} = 53°$, $\hat{B} = 37°$ and $\hat{C} = 90°$.

c) $\triangle ABC$ is a right-angled scalene triangle.

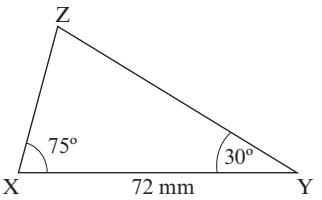
3. a)



b) $\hat{S} = 22.5°$, $\hat{T} = 22.5°$ and ST = 83 mm.

c) $\triangle RST$ is an obtuse-angled isosceles triangle.

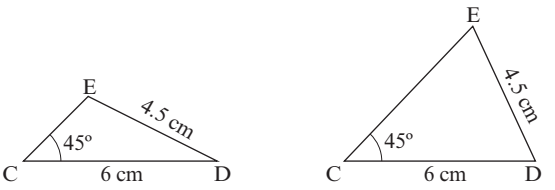
4. a)



b) $\hat{Z} = 75°$, YZ = 72 mm and XZ = 38 mm.

c) $\triangle XYZ$ is an acute-angled isosceles triangle.

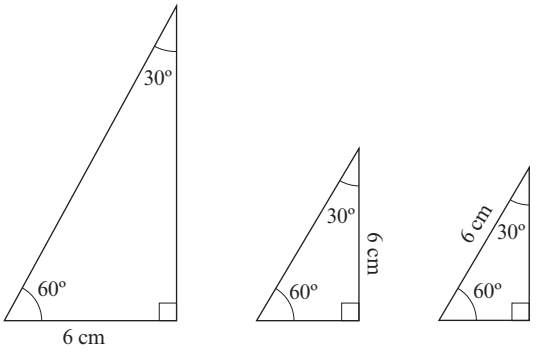
5. a)



b) In the first triangle: $\hat{D} = 26°$, $\hat{E} = 109°$ and CE = 2.8 cm.
In the second triangle: $\hat{D} = 64°$, $\hat{E} = 71°$ and CE = 5.7 cm.

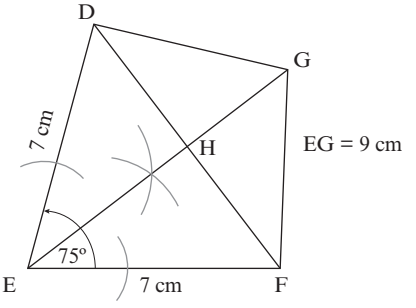
c) The first triangle is an obtuse-angled scalene triangle. The second triangle is an acute-angled scalene triangle.

6.



Exercise 13.5

1. a) – c)



d) DG = FG = 5.5 cm

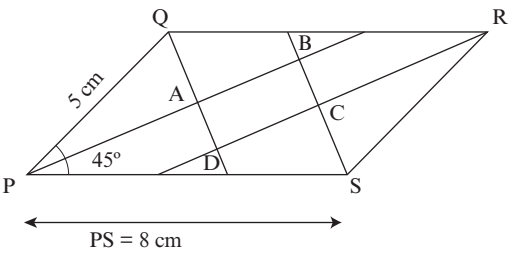
e) DEFG is a kite, because two pairs of adjacent sides are equal.

f) DH = FH = 4.3 cm and $D\hat{H}G = 90°$. DEFG is a kite, because the diagonals are perpendicular and one of the diagonals bisects the other.

2. a) and **b)** Student's own work.

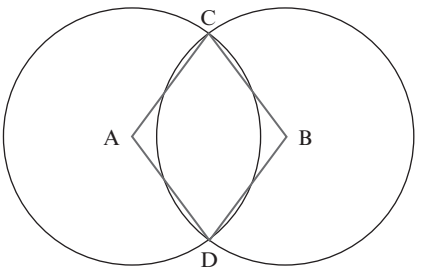
c) PQRS is a rectangle. Students can find this by measuring the lengths of the sides of the rectangle, or the angles of the rectangle.

3. a) – c)



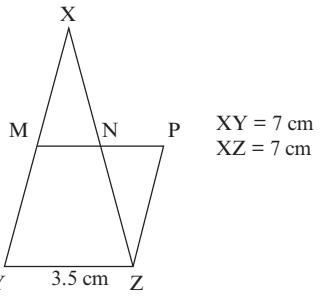
ACBD is a rectangle. Students can find this by measuring the lengths of the sides of the rectangle, or the angles of the rectangle.

4. a) – d)



e) ACBD is a rhombus with sides of 5 cm.

5. a) – c)



d) MPZY is a rhombus with sides of 3.5 cm.

6. a) – c) Student's own work.

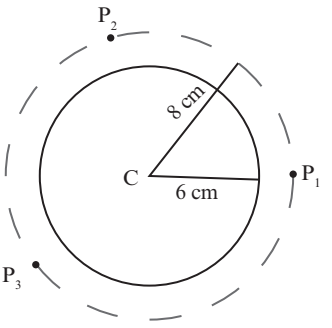
d) Students should find that $\hat{A} + \hat{C} = 180°$.

e) Students should find that $\hat{B} + \hat{D} = 180°$.

f) The two pairs of opposite angles of quadrilateral ABCD are supplementary.

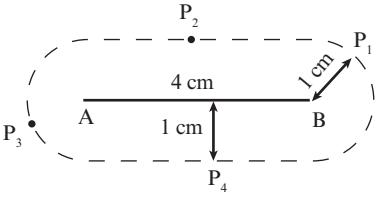
Exercise 13.6

1. a)



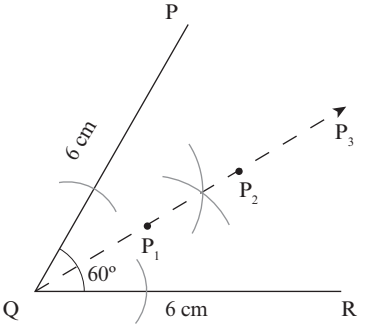
b) The locus of point P is the circle with centre C and a radius of 8 cm.

2. a)



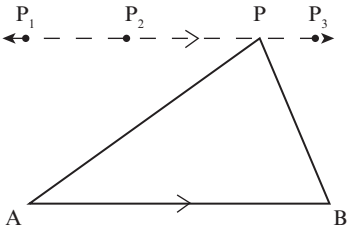
b) The locus of point P consists of two parallel lines and two semicircles. The lines are 1 cm away from the line AB and the radius of each semicircle is 1 cm.

3. a) and **b)**



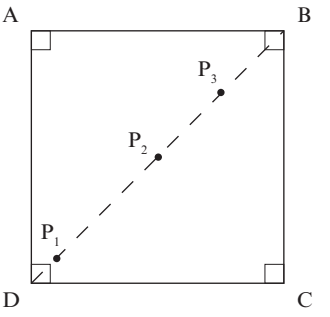
c) The locus of point P is the bisector of $P\hat{Q}R$.

4. a) and **b)**



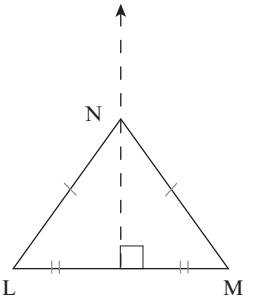
c) The locus of point P is an infinitely long straight line through point P, parallel to AB.

5. a) and **b)**



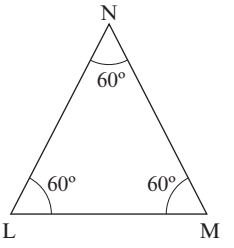
c) The locus of point P is the diagonal BD.

6. a)



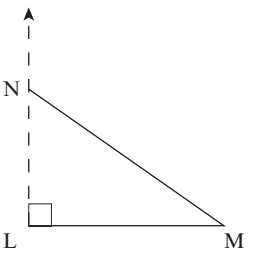
The locus of point N is the perpendicular bisector of LM.

b)



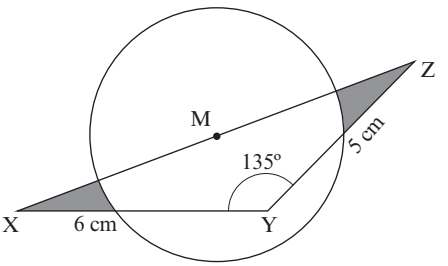
The locus of point N is the point such that NL = LM = MN.

c)



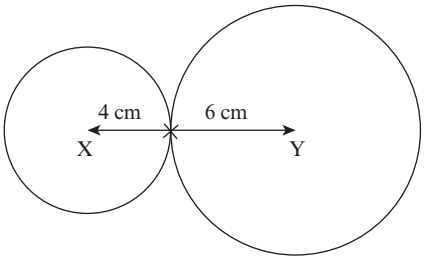
The locus of point N is the line NL where NL ⊥ LM.

7. a) – d)

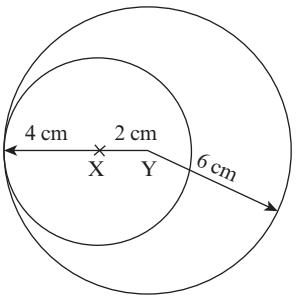


8. a) The locus of point P is a circle with centre X and radius 4 cm.
The locus of point Q is a circle with centre Y and radius 6 cm.

b) If the loci touch but do not overlap, the distance between X and Y must be 10 cm, as shown in the diagram below.



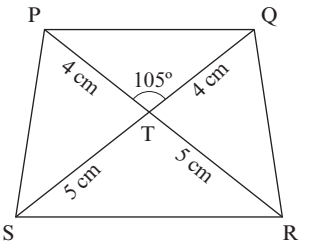
c) If the loci do not touch or overlap, the distance between X and Y must be greater than 10 cm. This can be seen in the diagram above. Alternatively, the smaller circle must lie inside the larger circle. So the distance between X and Y must be less than 2 cm. The diagram shows that the loci will touch if the distance between and X and Y is 2 cm.



Assess your progress

1. and **2.** Student's own work. Check each student's construction.

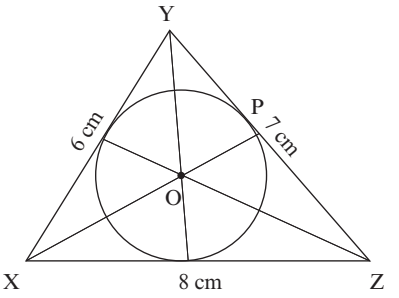
3. a) – b)



c) $Q\hat{P}S = 99°$, $P\hat{Q}R = 99°$, $Q\hat{R}S = 81°$ and $P\hat{S}R = 81°$.

d) PQRS is an isosceles trapezium. PQ || SR, because cointerior angles are supplementary.

4. a) – c)

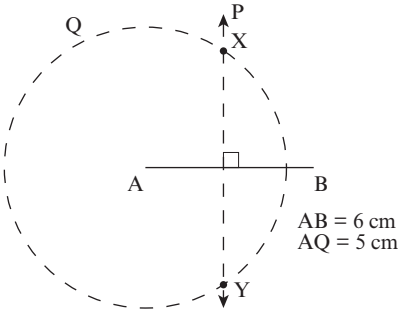


d) The circle touches all three sides of the triangle.

5. a) Points P and Q both fall on the perpendicular bisector of MN.

b) MNPQ is a kite.

6. a) – c)



d) The loci of P and Q intersect in two points, indicated by X and Y in the construction above. The distance between these points is 8 cm.

TOPIC 14

Triangles, parallel lines, parallelograms

Introduction

In this topic we begin with triangles, and revise types of angles and relationships between angles.

We move on to dealing with the properties of triangles, and revise how to set out formal proofs of theorems, by proving that angles on a straight line are supplementary.

Then we carry out the proofs that students need to focus on in this topic:

- The sum of the angles of a triangle is 180°.
- Any exterior angle of a triangle is equal to the sum of the two opposite interior angles.

We then address the different cases of congruence and how to prove that two triangles are congruent.

The next section deals with parallel lines and the properties of parallelograms, focusing on riders. We learn the properties of angles between parallel lines cut by a transversal. Then we revise properties of proportionality of triangles and learn how to prove and apply the intercept theorem.

The following section addresses parallelograms, and we carry out two proofs on parallelograms.

Finally we explore the concept of polygons, and learn about different types.

Common difficulties

Students confuse different types of angles and need to learn the names of the different sized angles thoroughly. They need to be able to draw a sketch of any size angle and recognise types of angles.

Make sure they understand all different types of triangles that are possible and how to calculate the interior angles. Many students often forget the very useful fact that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Students may be confused about which case of congruence to use when proving geometry riders. Give them many examples

of the four different types of congruence. Make sure that students know the difference between corresponding angles that are equal and cointerior angles that are supplementary.

Make sure that students know the difference between congruence and similarity. Sometimes students use incorrect lengths when calculating ratios of sides. Explain that the entire side of a triangle must be taken when comparing ratios of sides of similar triangles.

Preparation

Have pictures of different triangles and parallelograms on the classroom walls.

Have a chart with the different cases of congruence illustrated with examples. Have protractors available for checking the size of angles.

Introduction for students

Students need to fully grasp the conventions and terminology of setting out a formal geometry proof. They also need to be able to demonstrate their deductive reasoning when using riders.

Revise different sizes of angles and the names for each type of angle with illustrations.

Discuss the difference between complementary angles and supplementary angles.

Work through the examples and revise vertically opposite angles.

Explain that the properties of angles, parallel lines, triangles and similar triangles will all be an important foundation for learning the intercept theorem later in this topic.

Students sometimes get different quadrilaterals confused and need to learn the properties of each parallelogram thoroughly.

Answers

Exercise 14.1

1. **a)** Right angle **b)** Obtuse angle **c)** Acute angle
d) Reflex angle

2. **a)** 72° **b)** 332° **c)** 278°
d) 232°

Exercise 14.2

1. $x = 130°$; $y = 50°$

2. **a)** 18° **b)** 35°
c) 150° **d)** 84°
e) 88° **f)** $r = 72° - c$; $c = 72° - r$
g) $x = 130°$; $y = 50°$ **h)** 75°

3. **a)** 5° **b)** 115° **c)** 55°

4. **a)** c and d are vertically opposite angles
b) x and y are vertically opposite angles

5. Only c) is true.

Exercise 14.3

1. **a)** 70° **b)** 30° **c)** 37°
d) 128°

2. **a)** $a = 60°$; $b = 60°$; $c = 60°$
b) $d = 60°$
c) $e = 80°$; $f = 50°$

3. **a)** $x = 30°$
b) $x = 70°$
c) $x = 44°$; $z = 106°$; $y = 150°$
d) $x = 142°$
e) $x = 80°$; $y = 100°$; $z = 100°$

Exercise 14.4

1. a) SAS **b)** SSS **c)** SAS

2. a) side, angle, side
b) right angle, hypotenuse, leg
c) angle, angle, side

3. a) $\triangle ABC$ and $\triangle FDE$
b) AAS

4. a) In $\triangle PQB$ and $\triangle RQC$:

$BQ = QC$	(given)
$PQ = QR$	(given)
$P\hat{Q}B = R\hat{Q}C$	(vert. opp. $\angle$s)
$\therefore \triangle PQB \equiv \triangle RQC$	

b) In ΔABR and ΔACP:

$BQ + QR = BR$	
$QC + QP = PC$	
$\therefore BR = PC$	
$\hat{A} = \hat{A}$	(common $\angle$)
$\hat{B} = \hat{C}$	($\triangle PQB \Vert\Vert\Vert \triangle RQC$)
$\therefore \triangle ABR \equiv \triangle ACP$	(AAS)

Exercise 14.5

1. a) supplementary **b)** vertically opposite
c) alternate **d)** supplementary
e) vertically opposite **f)** corresponding

2. $a = 60°$, $b = 120°$, $c = 60°$, $w = 120°$, $x = 60°$, $y = 120°$, $z = 60°$

3. a) $a = 95°$ (alt. $\angle$s) **b)** $b = 100°$ (alt. $\angle$s)
c) $c = 85°$ (corr. $\angle$s) **d)** $d = 75°$ (corr. $\angle$s)

4.

$P\hat{Q}T + T\hat{Q}S = 180°$	($\angle$s on straight line PQS)
$P\hat{Q}R + R\hat{Q}S = 180°$	($\angle$s on straight line PQS)
and $T\hat{Q}S = x = R\hat{Q}S$	(given)
$\therefore P\hat{Q}T = P\hat{Q}R$	

5. $A\hat{B}D = A\hat{B}C - D\hat{B}C$
$= 90^o - D\hat{B}C$
$C\hat{B}E = D\hat{B}E - D\hat{B}C$
$= 90^o - D\hat{B}C$
$\therefore A\hat{B}D = C\hat{B}E$

6. $\triangle CDE \equiv \triangle FCG$ (given)
$\therefore C\hat{E}D = F\hat{G}C$ (from congruency)
$\therefore DE \parallel FG$ (corr. $\angle$s are equal)

Exercise 14.6

1. EF = 6 cm

2. AE = 7.5

3. CN = 2.8

4. WV = 11.7

5. IJ = 9 cm and KJ = 8 cm

6. 8 000 m

7. $y = 5$

8. GF = 12

9. BC = 10, CF = 15, CD = 9, CE = 6, EF = 9 and $\frac{DE}{AC} = \frac{3}{5}$

Exercise 14.7

1. a) 72° **b)** 135° **c)** 60°

2. a) 720° **b)** 120°

3. a) 20 **b)** 8 **c)** 60

4. a) 12 **b)** 20 **c)** 15

5. a) In $\triangle MNR$, $\hat{N}_1 = \hat{R}_2 = 58°$ (base $\angle$s, isosceles $\triangle$)
Similarly, in $\triangle SQP$, $\hat{Q}_1 = \hat{P} = 64°$ (base $\angle$s, isosceles $\triangle$)
$\therefore \hat{R}_1 = 122°$ (ext. $\angle$ = sum int. opp. $\angle$s)
and $\hat{S}_1 = 128°$ (ext. $\angle$ = sum int. opp. $\angle$s)
$\therefore (2x + 6) + 2x + 122° + 128° = 360°$ (sum $\angle$s quad RNSQ)
$\therefore 4x = 104°$
$\therefore x = 26°$

b) There are a number of ways in which the students may prove that MNPQ is a parallelogram. One method is set out here.
$R\hat{Q}S = 52°$
and $Q\hat{S}P = 52$ (given)
$\therefore$ RQ || SP (alt. $\angle$s equal)
Also $R\hat{Q}P = 52° + 64° = 116°$
and $\hat{M} = 64$ (given)
$\therefore$ MN || QP (coint. $\angle$s supplementary)
$\therefore$ MNPQ is a parallelogram (both pairs opp. sides ||)

6. $H\hat{E}J = H\hat{G}J$ (opp. $\angle$s parm)
In $\triangle HEF$ and $\triangle DGJ$:
1) $H\hat{E}F = D\hat{G}J$ ($H\hat{E}J + x = H\hat{G}J + x$)
2) HE = GJ (opp. sides parm)
3) $EHJ = H\hat{J}G$ (alt. $\angle$s, HE || GJ)
$\therefore \triangle HEF \equiv \triangle DGJ$ (AAS)
$\therefore$ DG = EF (from congruency)
and $E\hat{F}H = G\hat{D}J$ (from congruency)
$\therefore$ DG || EF (alt. $\angle$s equal)
$\therefore$ DEFG is a parallelogram (one pair opp. sides equal and ||)

7. a) In $\triangle AOB$ and $\triangle POC$:
1) $A\hat{B}O = P\hat{C}O$ (alt. $\angle$s, AB || PC)
2) BO = OC (given)
3) $A\hat{O}B = P\hat{O}C$ (vert. opp. $\angle$s)
$\therefore \triangle ABO \equiv \triangle PCO$ (AAS)
b) AB = PC (from congruency)
$\therefore$ ABPC is a parallelogram (one pair opp. sides equal and ||)

Assess your progress

1. **a)** $x = 60°$ **b)** $x = 60°$ **c)** $x = 35°$
d) $x = 40°$

2. **a)** $\triangle ABC \equiv \triangle EDC$ (SAS)
b) $\triangle ABC \equiv \triangle DBC$ (RHL or AAS)
c) Triangles are not necessarily congruent.
d) Triangles are not congruent.
e) $\triangle ABC \equiv \triangle ADC$ (AAS)

3. OP and QR are not parallel as the cointerior angles do not add up to 180°.

4. Use proof in Student's Book as a guide.

5. AE = 4 cm, EC = 24.5 cm and BE = 5.5 cm

6. MR = 12, RP = 6, QN = 4 and MN = 12

7. Use proof in Student's Book as a guide.

8. $\frac{DS}{SZ} = \frac{3}{1}$

9. 12 sides

10. 3.27 m

11. $\frac{h}{0.05} = \frac{90}{1}$

$\therefore h = 4.5$ m

12. 3 980 m

13. Use proof in Student's Book as a guide.

TOPIC 15

Statistics

Introduction

In this topic, your students will build on their existing knowledge of statistics as they calculate measures of central tendency and spread, organise data in frequency tables, present data in a variety of charts, and interpret data. In previous years your students have worked only with ungrouped data. In SS1 they will revise what they already know, and then will start working with grouped data.

Common difficulties

Students often enjoy working with statistics, as it provides a welcome change of pace from 'normal' mathematics. However, they may find some of the concepts confusing and will need careful support when it comes to interpreting some of the more subtle frequency tables. Some students also feel intimidated by questions that require them to interpret data, given certain statistics about the data.

Preparation

Find examples of different statistical charts and display these on the walls of your classroom.

Look for real examples of pie charts, line graphs, bar graphs, compound bar graphs, histograms and frequency polygons. Do not worry about including bar graphs with horizontal bars or any other examples of unusual graphs.

It is important that your students realise that statistical graphs are widely used and that they will not always look exactly like the graphs that they will learn about in school.

Where students are expected to work in groups or pairs on exercises, facilitate these interactions.

Introduction for students

Write the following terms on the board:

mean	mode
median	frequency table
tallies	pie chart

These are all terms that your students learnt about in JS1 to JSS3. Ask a volunteer from your class to choose any term and then explain it to the class. Once the correct explanation has been given, cross the term out and ask a new volunteer to choose one of the remaining terms and explain it to the class. Continue until all the terms have been discussed.

Answers

Exercise 15.1

1. a) This data set has already been ordered.

(i) Mean $= \frac{3 + 8 + 9 + 17 + 35 + 38 + 39 + 42 + 44}{9}$

$= 26.11$ (correct to 2 d.p.)

(ii) There are 9 data values in the data set. The middle value is the 5th value, which is 35. Median = 35.

(iii) There is no mode, because no data value occurs more than once.

(iv) Range = 44 – 3 = 41

b) Ordered data set: {24, 24, 31, 33, 42, 56, 57, 60, 60, 70, 81, 84, 94, 97}

(i) Mean

$= \frac{24 + 24 + 31 + 33 + 42 + 56 + 57 + 60 + 60 + 70 + 81 + 84 + 94 + 97}{14}$

$= 58.07$ (correct to 2 d.p.)

(ii) There are 14 data values in the data set. The middle value lies halfway between the 7th and the 8th data values, which are 57 and 60. $\frac{57 + 60}{2} = 58.5$.
Median = 58.5.

(iii) There are two modes: 24 and 60.

(iv) Range = 97 – 24 = 73.

c) Ordered data set: {–23, –16, –13, –7, –7, 3, 4, 5, 5, 10, 11, 14, 18}

(i) Mean = $\frac{-23 - 16 - 13 - 7 - 7 + 3 + 4 + 5 + 5 + 10 + 11 + 14 + 18}{13}$

= 0.31 (correct to 2 d.p.)

(ii) There are 13 data values in the data set. The middle value is the 7th value, which is 4. Median = 4.

(iii) There are two modes: –7 and 5.

(iv) Range = 18 – (–23) = 41.

d) Ordered data set: {–8.5, –5.2, –1.1, 0.3, 5.1, 5.9, 7.5, 7.5, 7.6, 8.4, 8.8, 9.9}

(i) Mean = $\frac{-8.5 - 5.2 - 1.1 + 0.3 + 5.1 + 5.9 + 7.5 + 7.5 + 7.6 + 8.4 + 8.8 + 9.9}{12}$

= 3.85

(ii) There are 12 data values in the data set. The middle value lies halfway between the 6th and the 7th data values, which are 5.9 and 7.5.

$\frac{5.9 + 7.5}{2}$ = 6.7. Median = 6.7.

(iii) The mode is 7.5.

(iv) Range = 9.9 – (–8.5) = 18.4.

2. The median tells the teacher that half of the class scored 28 marks or less, and the other half of the class scored 28 marks or more.

3. The range tells the teacher that the difference between the highest and lowest marks is 42.

Exercise 15.2

1.

Type of vehicle	Tally	Frequency
Buses	𝍸 IIII	9
Cars	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II	37
Taxis	𝍸 𝍸 𝍸	15
Other	𝍸 𝍸 II	12

2. a)

No. of glasses of water	Tally	Frequency
0	III	3
1	𝍸 III	8
2	𝍸 III	8
3	II	2
4	I	1
5	𝍸 I	6
6	III	3
7	II	2
8	II	2

b) (i) The sum of all the data values is $(0 \times 3) + (1 \times 8) + (2 \times 8) + (3 \times 2) + (4 \times 1) + (5 \times 6) + (6 \times 3) + (7 \times 2) + (8 \times 2) = 112$.

Mean $= \frac{112}{35} = 3.2$.

(ii) There are 35 data values in the data set. The middle value is the 18th value, which is 2. Median = 2.

(iii) There are two modes: 1 and 2.

(iv) Range = 8 – 0 = 8.

c) There are 35 students altogether. Of these, 14 students drink at least four glasses of water per day. $\frac{14}{35} \times 100\% = 40\%$. So 40% of the students drink at least four glasses of water per day.

Exercise 15.3

This Exercise is based on the example on pages 247 and 248 of the Student's Book.

1. Student's own work.

2. Table 3

3. Table 1

4. 70% of 30 = 21, so the table(s) must show how many students obtained 21 marks or more. Tables 1 and 2 show this information.

5. The table(s) must show how many students obtained 9 marks or less. Table 3 shows this information.

Exercise 15.4

1. 84 – 45 = 39

a) 39 ÷ 5 = 7.8
Rounded up to the nearest unit, the class interval is 8. This gives us the following five classes: 45–52, 53–60, 61–68, 69–76, and 77–84.

b) 39 ÷ 8 = 4.875
Rounded up to the nearest unit, the class interval is 5. This gives us the following eight classes: 45–49, 50–54, 55–59, 60–64, 65–69, 70–74, 75–79 and 80–84.

2. 108 kg – 56 kg = 52 kg

a) 52 kg ÷ 6 = 8.66… kg
Rounded up to the nearest unit, the class interval is 9 kg. This gives us the following six classes: 56–64 kg, 65–73 kg, 74–82 kg, 83–91 kg, 92–100 kg and 101–109 kg.

b) 52 kg ÷ 7 = 7.42… kg
Rounded up to the nearest unit, the class interval is 8 kg. This gives us the following seven classes: 56–63 kg, 64–71 kg, 72–79 kg, 80–87 kg, 88–95 kg, 96–103 kg and 104–111 kg.

3. 1.98 m – 0.36 m = 1.62 m

a) 1.62 m ÷ 4 = 0.405 m
Rounded up to two decimal places, the class interval is 0.41 m. This gives us the following four classes: 0.36–0.76 m, 0.77–1.17 m, 1.18–1.58 m and 1.59–1.99 m.

b) 1.62 m ÷ 8 = 0.2025 m
Rounded up to two decimal places, the class interval is 0.21 m. This gives us the following eight classes: 0.36–0.56 m, 0.57–0.77 m, 0.78–0.98 m, 0.99–1.19 m, 1.20–1.40 m, 1.41–1.61 m, 1.62–1.82 m and 1.83–2.03 m.

Exercise 15.5

1.

Mass (kg)	40–49	50–59	60–69	70–79	80–89
Lower class boundary (kg)	39.5	49.5	59.5	69.5	79.5
Upper class boundary (kg)	49.5	59.5	69.5	79.5	89.5

2.

Time (s)	15–18	19–22	23–26	27–29	30–33
Lower class boundary (s)	14.5	18.5	22.5	26.5	29.5
Upper class boundary (s)	18.5	22.5	26.5	29.5	33.5

3.

Capacity (ℓ)	1.1–1.5	1.6–2.0	2.1–2.5	2.6–3.0	3.1–3.5
Lower class boundary (ℓ)	1.05	1.55	2.05	2.55	3.05
Upper class boundary (ℓ)	1.55	2.05	2.55	3.05	3.55

Exercise 15.6

1.

Mass (kg)	40–49	50–59	60–69	70–79	80–89
Midpoint (kg)	44.5	54.5	64.5	74.5	84.5

2.

Capacity (ℓ)	1.1–1.5	1.6–2.0	2.1–2.5	2.6–3.0	3.1–3.5
Midpoint (ℓ)	1.3	1.8	2.3	2.8	3.3

3.

Time (s)	15–18	19–22	23–26	27–30	31–34
Midpoint (s)	16.5	20.5	24.5	28.5	32.5

Exercise 15.7

1. a)

Height (cm)	Tally	Frequency
[43.0, 46.0)	𝍸 𝍸	10
[46.0, 49.0)	𝍸 IIII	9
[49.0, 52.0)	𝍸 𝍸 𝍸	15
[52.0, 55.0)	𝍸 II	7
[55.0, 58.0)	𝍸 III	8

b) 15 goats have a height of 52 cm or more. $\frac{15}{49} \times 100\% = 30.6\%$. So 30.6% of the goats have a height of 52 cm or more.

2. a)

Cholesterol level	Tally	Frequency
171–180	𝍸 I	6
181–190	𝍸 III	8
191–200	𝍸	5
201–210	II	2
211–220	III	3
221–230	𝍸	5
231–240	𝍸 I	6
241–250	𝍸	5

b) 21 patients had a cholesterol level of more than 200. $\frac{21}{40} \times 100\% = 52.5\%$. So 52.5% of the patients had an undesirable cholesterol level.

Exercise 15.8

1. a) (i) First calculate the midpoints of the classes:

Number of tourists	0–9	10–19	20–29	30–39	40–49
Midpoint	4.5	14.5	24.5	34.5	44.5
Frequency	11	9	13	13	8

The estimated sum of all the data values is $(4.5 \times 11) + (14.5 \times 9) + (24.5 \times 13) + (34.5 \times 13) + (44.5 \times 8) = 1\,303$. There are $(11 + 9 + 13 + 13 + 8) = 54$ data values.

The estimated mean $= \frac{1\,303}{54} = 24$ tourists (correct to the nearest whole number).

(ii) There are 54 data values.

The median is halfway between the 27th and the 28th data value. Median = the 27.5th data value.

The 27.5th data value is the 7.5th data value in the class 20–29 (because $27.5 - (11 + 9) = 7.5$).

The estimated median $= 19.5 + \frac{7.5}{13} \times 10 = 25$ tourists (correct to the nearest whole number).

b) There are two modal classes, 20–29 and 30–39, because they both have the highest frequency.

c) The largest possible range = 49 – 0 = 49 tourists. The smallest possible range = 40 – 9 = 31 tourists. So all we can say for certain is that the range lies between 31 and 49 tourists.

2. a) (i) First calculate the midpoints of the classes:

Width (cm)	(3.2, 3.6]	(3.6, 4.0]	(4.0, 4.4]	(4.4, 4.8]	(4.8, 5.2]	(5.2, 5.6]
Midpoint	3.4	3.8	4.2	4.6	5.0	5.4
Frequency	5	8	12	14	5	2

The estimated sum of all the data values is $(3.4 \times 5) + (3.8 \times 8) + (4.2 \times 12) + (4.6 \times 14) + (5.0 \times 5) + (5.4 \times 2) = 198$.

There are $(5 + 8 + 12 + 14 + 5 + 2) = 46$ data values.

The estimated mean $= \frac{198}{46} = 4.3$ cm (correct to the nearest mm).

(ii) There are 46 data values.

The median is halfway between the 23rd and the 24th data value, Median = the 23.5th data value.

The 23.5th data value is the 10.5th data value in the class (4.0, 4.4] (because $23.5 - (5 + 8) = 10.5$).

The estimated median $= 4.0 + \frac{10.5}{12} \times 0.4 = 4.4$ cm (correct to the nearest mm).

b) The modal class is (4.4, 4.8], because it has the highest frequency.

c) The largest possible range = 5.6 – 3.2 = 2.4 cm. The smallest possible range = 5.2 – 3.6 = 1.6 cm. So all we can say for certain is that the range lies between 1.6 cm and 2.4 cm.

Exercise 15.9

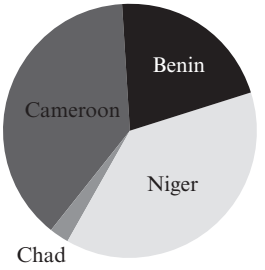
1. a) 773 + 1 497 + 87 + 1 690 = 4 047 km

b)

Neighbouring country	Land border (km)	Percentage of total land border	Sector angle
Benin	773	19.1%	68.8°
Niger	1 497	37.0%	133.2°
Chad	87	2.1%	7.7°
Cameroon	1 690	41.8%	150.3°
Total	**4 047**	**100.0%**	**360°**

c)

Land borders of Nigeria with neighbouring countries



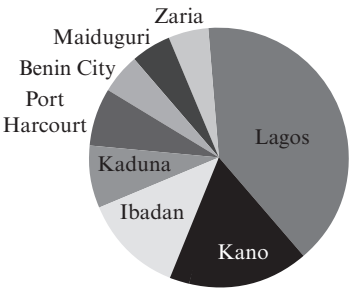
2. a) 7 937 932 + 3 848 885 + 3 078 400 + 1 652 844 + 1 320 214 + 1 051 600 + 1 044 497 + 1 018 827 = 20 953 199

b)

City	Population	Percentage of total population	Sector angle
Lagos	7 937 932	37.9%	136.4°
Kano	3 848 885	18.3%	66.1°
Ibadan	3 078 400	14.7%	52.9°
Kaduna	1 652 844	7.9%	28.4°
Port Harcourt	1 320 214	6.3%	22.7°
Benin City	1 051 600	5.0%	18.1°
Maiduguri	1 044 497	5.0%	17.9°
Zaria	1 018 827	4.9%	17.5°
Total	**20 953 199**	**100.0%**	**360°**

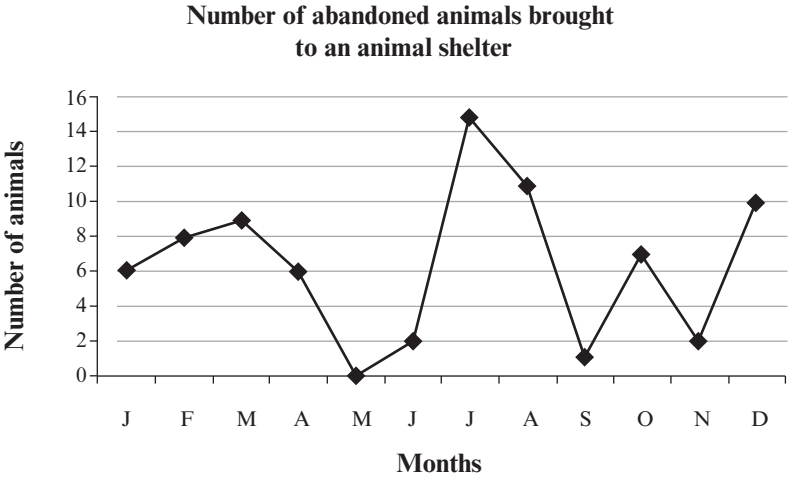
c)

Population of eight major cities in Nigeria



Exercise 15.10

1. a)



b) Students should notice that there is a connection between the dates of the school holidays and the numbers of abandoned animals. The longest school holiday occurs in July and August, which coincides with the two highest points on the graph. The other high points occur in October, December, February, March and April, all of which coincide with the school holidays. This seems to imply that some people who go away on holiday abandon their animals instead of making arrangements for them to be taken care of.

2. a)

b) The inflation rate was highest in 2001.

c) The inflation rate was lowest in 2007.

d) The increase in the inflation rate was greatest between 2007 and 2008.

e) The decrease in the inflation rate was greatest between 2005 and 2006.

f) Students should comment that there has been a gradual overall decrease in the inflation rate.

Exercise 15.11

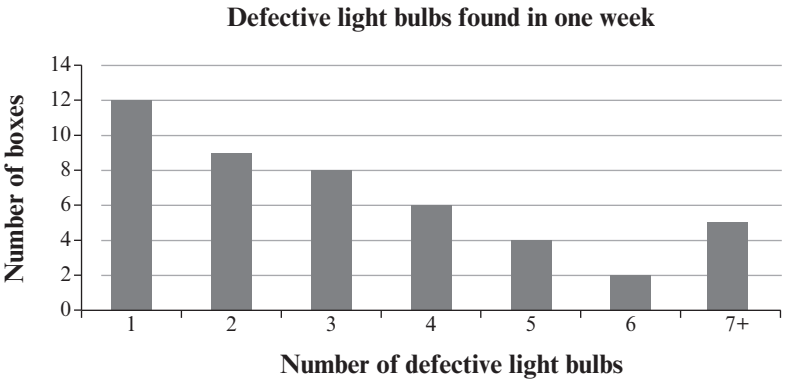
1. To decide this question, we need to calculate the average number of questions that the girls answered correctly and compare that to the average number of questions that the boys answered correctly.
The girls answered $(0 \times 3) + (1 \times 2) + (2 \times 6) + (3 \times 9) + (4 \times 3) + (5 \times 3) = 68$ questions correctly.
There were $(3 + 2 + 6 + 9 + 3 + 3) = 26$ girls in the class.
$68 \div 26 = 2.62$ (correct to 2 d.p.)
So on average, the girls answered 2.62 questions correctly.
The boys answered $(0 \times 1) + (1 \times 4) + (2 \times 6) + (3 \times 6) + (4 \times 5) + (5 \times 2) = 64$ questions correctly.
There were $(1 + 4 + 6 + 6 + 5 + 2) = 24$ boys in the class.
$64 \div 24 = 2.67$ (correct to 2 d.p.)
So on average, the boys answered 2.67 questions correctly.
So the boys performed slightly better than the girls.

2. a) $12 + 9 + 8 + 6 + 4 + 2 + 5 = 46$. So 46 boxes contained defective light bulbs.

b) $\frac{46}{500} \times 100\% = 9.2\%$. So 9.2% of the 500 boxes that were tested contained defective light bulbs.

c) The '7+ ' means '7 or more'.

d)



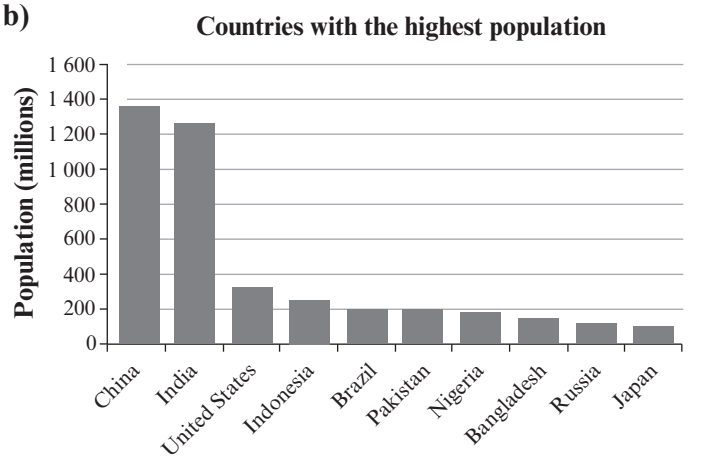
e) The most common number of defective light bulbs in the boxes represented in the table was 1.

f) The most common number of defective light bulbs in the 500 boxes that were tested was 0, because most of the boxes contained no defective light bulbs.

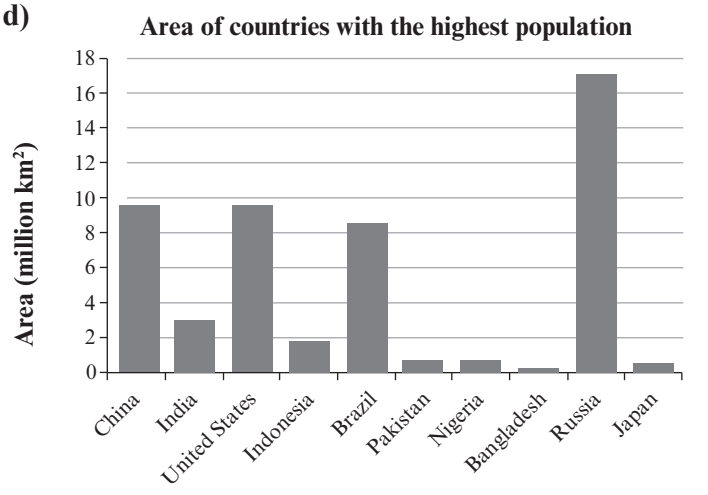
3. a), c), e)

Rank	Country	Population (2014) (in millions)	Area (in millions of km^2)	Population density (population/km^2)
1	China	1 356	9.6	141.3
2	India	1 236	3.2	386.3
3	United States	319	9.5	33.6
4	Indonesia	254	1.9	133.7
5	Brazil	203	8.5	23.9
6	Pakistan	196	0.9	217.8
7	Nigeria	177	0.9	196.7
8	Bangladesh	166	0.1	1 660.0
9	Russia	142	17.1	8.3
10	Japan	127	0.4	317.5

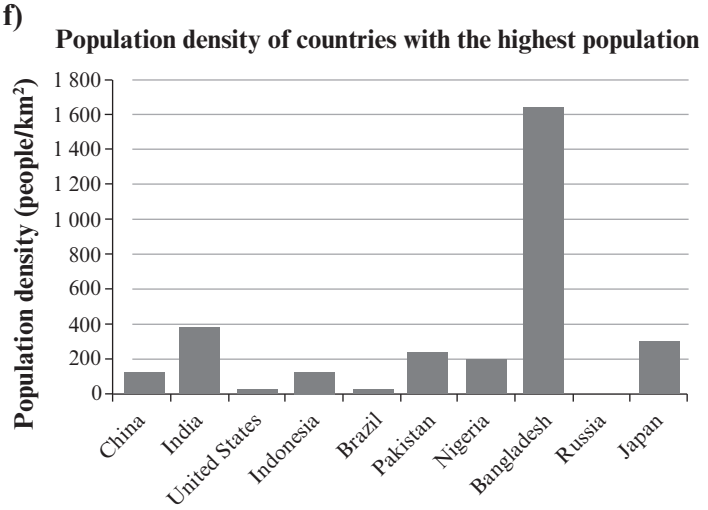
b)



d)



f)

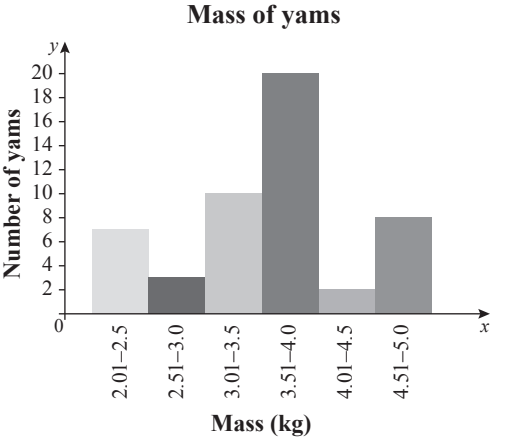


g) (i) Russia has the greatest area and the smallest population density.

(ii) Bangladesh has the smallest area and the greatest population density.

Exercise 15.12

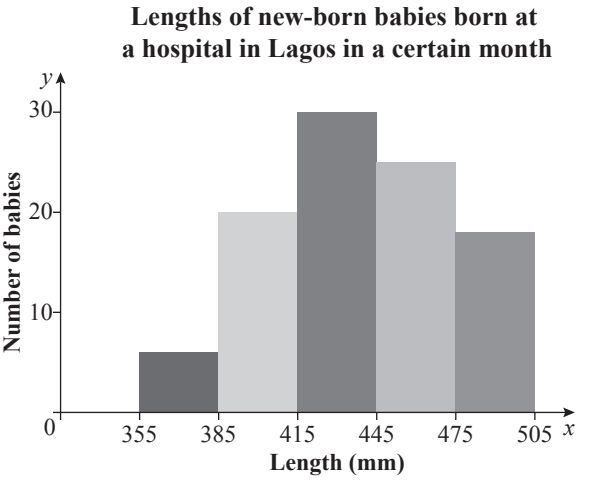
1.



2. a) The notation (355, 385] means all the lengths between 355 mm and 385 mm. The round bracket means that a length of 355 mm is not included in this interval. The square bracket means that a length of 385 mm is included in this interval.

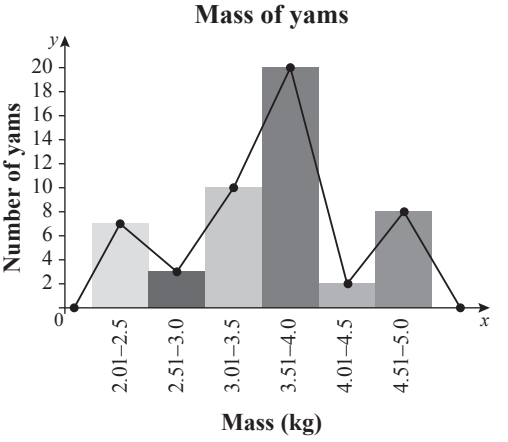
b) 100 babies were born in the hospital during that week.

c)

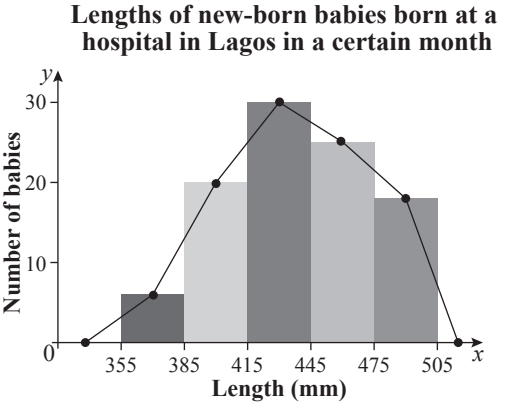


Exercise 15.13

1.



2.



Assess your progress

1. 36.5 °C – 25.3 °C = 11.2 °C

a) 11.2 °C ÷ 5 = 2.24 °C
Class interval is 2.3 °C (to one decimal place). This gives us the following five classes: 25.3–27.5 °C, 27.6–29.8 °C, 29.9–32.1 °C, 32.2–34.4 °C and 34.5–36.7 °C.

b) 11.2 °C ÷ 9 = 1.24444… °C
Class interval is 1.3 °C (to one decimal place). This gives us the following nine classes: 25.3–26.5 °C, 26.6–27.8 °C, 27.9–29.1 °C, 29.2–30.4 °C, 30.5–31.7 °C, 31.8–33.0 °C, 33.1–34.3 °C, 34.4–35.6 °C and 35.7–36.9 °C.

2.

Distance (km)	6–9	10–13	14–17	18–21	22–25
Lower class boundary (km)	5.5	9.5	13.5	17.5	21.5
Upper class boundary (km)	9.5	13.5	17.5	21.5	25.5
Midpoint (km)	7.5	11.5	15.5	19.5	23.5

3. a) (i) First calculate the midpoints of the classes:

Time (minutes)	(0, 2]	(2, 4]	(4, 6]	(6, 8]	(8, 10]	(10, 12]
Midpoint	1	3	5	7	9	11
Frequency	16	33	69	50	24	9

The estimated sum of all the data values is (1 × 16) + (3 × 33) + (5 × 69) + (7 × 50) + (9 × 24) + (11 × 9) = 1 125. There are (16 + 33 + 69 + 50 + 24 + 9) = 201 data values.

The estimated mean = $\frac{1\,125}{201}$ = 5.60 min (correct to 2 d.p.)

(ii) There are 201 data values. The median is the 101st data value.

The 101st data value is the 52nd data value in the class (4, 6] (because 101 – (16 + 33) = 52).

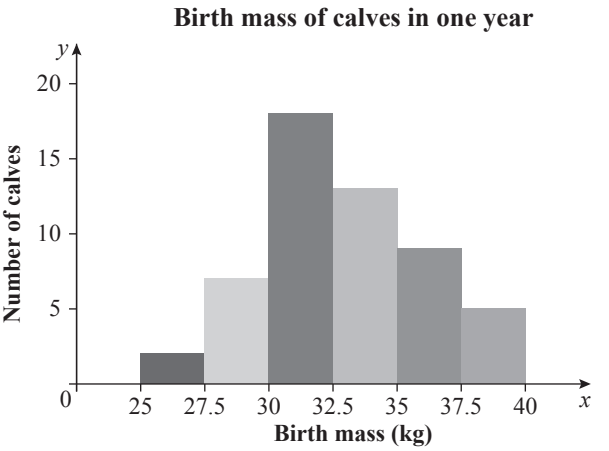
The estimated median = 4 + $\frac{52}{69}$ × 2 = 5.51 min (correct to 2 d.p.)

b) The modal class is (4, 6], because it has the highest frequency.

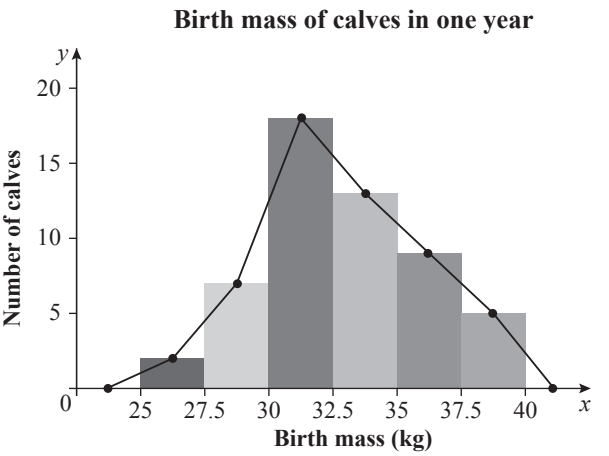
c) The largest possible range = 12 – 0 = 12 min. The smallest possible range = 10 – 2 = 8 min. So all we can say for certain is that the range lies between 8 min and 12 min.

4. a) A calf with a birth mass of 35.0 kg would fall in the class [35, 37.5).

b)



c)



TOPIC 16

Revision and exam practice

1 Number and numeration

Exercise 16.1

1. A	**2.** C	**3.** B	**4.** D
5. B	**6.** B	**7.** C	**8.** B
9. C	**10.** A	**11.** C	**12.** A
13. B	**14.** D	**15.** B	**16.** C
17. B	**18.** D	**19.** A	**20.** D
21. B	**22.** D	**23.** B	**24.** A
25. A	**26.** C	**27.** B	**28.** C
29. A	**30.** B		

Exercise 16.2

1. a) 300_{10} **b)** 414_{10} **c)** $1\ 358_{10}$

2. a) $33\ 122_4$ **b)** $12\ 421_5$ **c)** $1\ 732_8$ **d)** 3DA

3. a) 398 000 000 **b)** 0.00000657

4. a) 7.993×10^{-6} **b)** 1.13050654×10^{8}

5. a) 4.92×10^{12} **b)** 3.993×10^{-2}

6. a) 1.40×10^{5} **b)** 1.76×10^{6}

7. a) 1.801×10^{5} **b)** -5.602×10^{-3}

8. a) 6^9 **b)** 3^{-3} **c)** 5^4 **d)** 10^4

9. a) $4a^8$ **b)** $4x$

10. a) $x = 3$ **b)** $x = 4$

11.a) -3 **b)** $\log_2 64 \times \log_8 4 = 4$

12. $\frac{1}{2}$

13. a) $A \cup B = \{1, 2, 3, 7, 8, 10, 14, 15\}$
b) $B \cap C = \{3, 7\}$
c) $(A \cup B)' = \{6, 13, 16, 18\}$
d) $B' \cap A' = \{6, 13, 16, 18\}$
e) $(B \cap C)' = \{1, 2, 6, 8, 10, 13, 14, 15, 16, 18\}$
f) $B' \cap C' = \{6, 8, 10, 14, 16, 18\}$

14. a) $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$
b) $B \cup A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17\}$
c) $(A \cup B)' = \{1, 5, 7, 11, 13, 17\}$
d) $B' \cap A' = \{1, 5, 7, 11, 13, 17\}$

2 Algebraic processes

Exercise 16.3

1. A **2.** C **3.** B **4.** D
5. A **6.** A **7.** D **8.** B
9. D **10.** D

Exercise 16.4

1. 2 years

2. $x = \frac{1}{2}$ and $y = -\frac{1}{2}$

3. a) $3(x^2 - 5x + 10)$ **b)** $(x - 4)(x - 8)$

c) $4(y + 3)(y + 2)$ **d)** $(x + 5)(x - 5)$

4. a) $x = 3$ or $x = -4$ **b)** $x = 5$ or $x = -1$

c) $x = 3$ or $x = -\frac{2}{3}$ **d)** $x = -3$ or $x = -\frac{1}{2}$

5. a) $x = 6.53$ or $x = -1.53$ **b)** $x = 1.79$ or $x = -2.79$

c) $x = -0.15$ or $x = -3.35$ **d)** $x = -1.43$ or $x = 2.10$

6. a) $x = 4.19$ or $x = -1.19$ **b)** $x = -4.56$ or $x = -0.44$

c) $x = 4.11$ or $x = -0.61$ **d)** $x = 2.39$ or $x = 0.28$

7. 14 and 16 or –14 and –16

8. a) $a = -1$, $b = 1$, and $c = 2$

b) $m = 1$ and $k = -2$

c) $x = -1$ or $x = 2$; Points of intersection: $(-2; -4)$ and $(2; 0)$

d) $x = \frac{1}{2}$

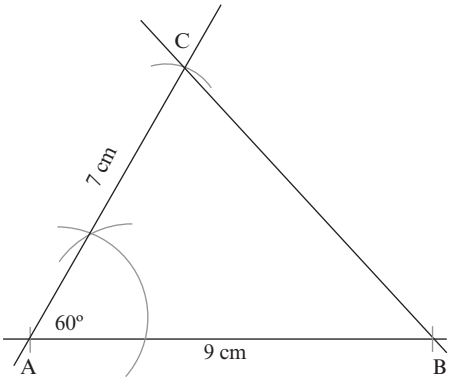
3 Geometry

Exercise 16.5

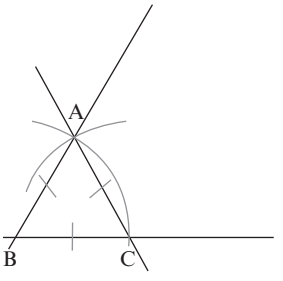
1. A	**2.** C	**3.** B	**4.** D
5. B	**6.** A	**7.** D	**8.** A
9. C	**10.** A	**11.** A	**12.** A
13. A	**14.** D	**15.** B	**16.** D
17. A	**18.** C	**19.** B	**20.** C

Exercise 16.6

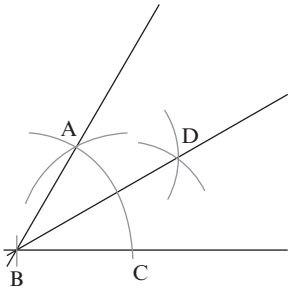
1.



2. a)



b)



3. a) (i) 1
(ii) –1
(iii) 1
(iv) –1

b) (i) 90°
(ii) 0° and 360°
(iii) 180°
(iv) 0°, 180° and 360°

4. SA of cone (circular base not included):
$\pi \times 6 \times 10 = 188.57\ cm^2$
SA of cylinder (circular bases not included):
$2 \times \pi \times 6 \times 12 = 452.57 cm^2$
SA of hemisphere:
$4 \times \pi \times 6^2 \times \frac{1}{2} = 226.29\ cm^2$
$\therefore$ TSA = $867.43\ cm^2$

5. $a = 107°$ (coint. ∠s suppl.)
$b = 73°$ (opp. ∠s in a parm =) or (coint. ∠s suppl.)
$c = 107°$ (coint. ∠s suppl.) or (opp. ∠s in a parm =)
$d = 73°$ (∠s on a straight line) or (corr. ∠s =)

6. a) ADCF is a parallelogram (diagonals bisect each other)
$\therefore$ AD || FC and AD = FC
$\therefore$ DB || FC and DB = FC (given AD = DB)
$\therefore$ DBCF is a parallelogram (one pair opp. sides = and ||)
$\therefore$ DE || BC

b) DE = EF (given)
DEF = BC (opp. sides parm)
$\therefore$ BC = 2DE

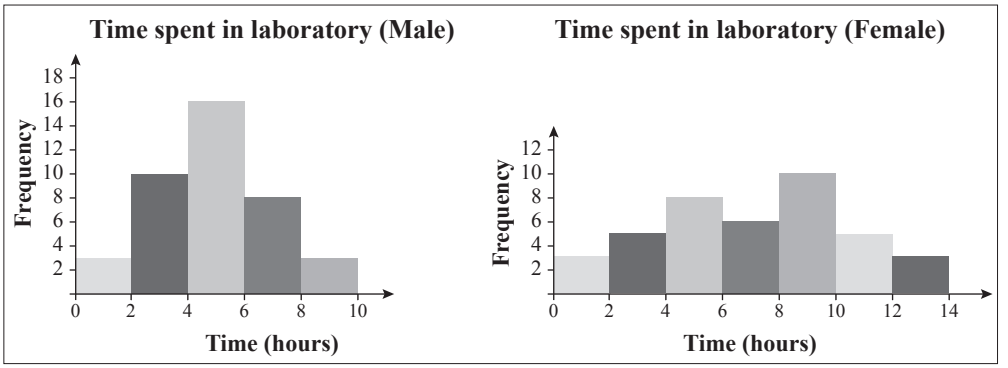
4 Statistics

Exercise 16.7

1. B **2.** A **3.** C **4.** D
5. B **6.** C **7.** C

Exercise 16.8

1. a)

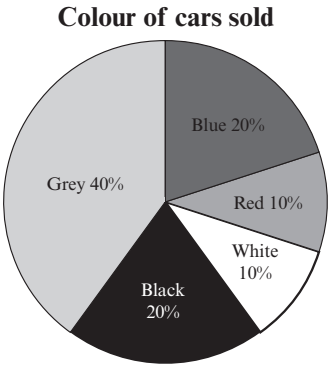


b)

	mean	median	mode
male students	5	4.8	$4 \leq t < 6$
female students	7	7.5	$8 \leq t < 10$

c) The mean and median suggest that female students use the laboratory for longer periods of time than the male students. The mode for female students is higher than for male students but the sample is too small for any meaningful conclusion.

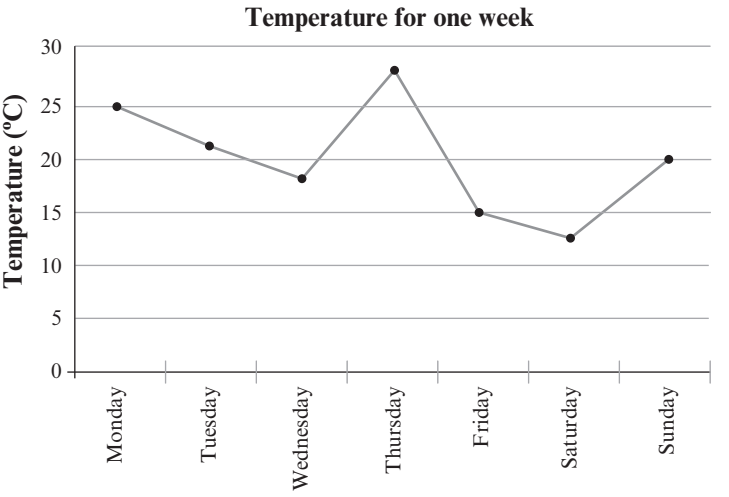
2.



Colour of car	Frequency	Angle	Percentage
Blue	2	$2 \times 36° = 72°$	$\frac{2}{10} \times 100 = 20\%$
Red	1	$1 \times 36° = 36°$	$\frac{1}{10} \times 100 = 10\%$
White	1	$1 \times 36° = 36°$	$\frac{1}{10} \times 100 = 10\%$
Black	2	$2 \times 36° = 72°$	$\frac{2}{10} \times 100 = 20\%$
Grey	4	$4 \times 36° = 144°$	$\frac{4}{10} \times 100 = 40\%$
	10	**360°**	**100%**

3. a) ₦8
b) ₦4

4. a)



b) 28 °C – 12 °C = 16 °C
c) 20 °C